Philosophy of Physics 1b – Lecture 2

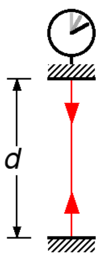
1 Review

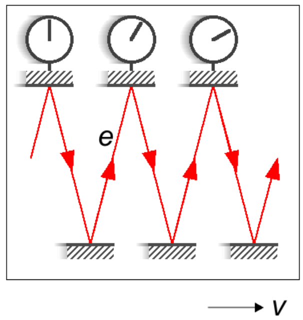
- **Galilean relativity:** the principle that "physics is the same" in all inertial (i.e. uniformly moving) frames of reference. (Recall Galileo's ship, fish, flies. etc.)
- Electromagnetic phenomena, including the propagation of light, seem to conflict with this principle – in the late C19, physicists (especially in Cambridge!) believed that the 'aether' in which EM waves were thought to travel provided an absolute 'rest frame'. (In Cambridge they seem to have thought that the aether was a challenge to materialism, and hence a refuge for spiritualism!)

2 Einstein

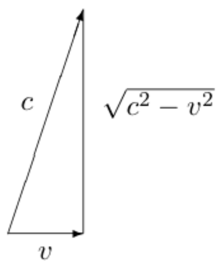
- Einstein wanted to combine Galilean relativity with Maxwell's theory of electromagnetism. Maxwell's theory implied a unique value for the speed of light, and so Einstein's explored the consequences of assuming that all uniformly moving observers would observe this same value for the speed of light.
- Some of the surprising consequences of this assumption emerge by considering a clock based on light.

3 Thinking about light clocks





- Q: What is a clock? A: A device for counting the repetitions of some regular process (pendulum swings, etc).
- Let's bounce light off a mirror, and use the time it takes for a round trip (i.e., $2d/c$) as the basis for a clock.
- Now put one of these clocks in a rocket, and send it at velocity v to the right. Notice what the path of the light ray looks like, from our point of view, as the rocket goes past us.
- According to Einstein's assumption, we still see the light ray travelling at the same speed (c) as in the stationary clock – but we see it travelling **further.** So we see the clock in the rocket *ticking slower* than the stationary clock.
- How much slower? We can figure that out by comparing the distance d across the stationary clock with the diagonal distance e on the moving clock. We want to know the ratio e/d. (This tells us by what proportion light has to travel further in the moving clock, and hence by what proportion the clock ticks slower.)



- Here Pythagorus (pictured right) is our friend. We want to know the ratio of the hypotenuse to the vertical side in the triangle shown. (Why is this triangle the one we need?)
- This ratio is $\frac{c}{\sqrt{c^2-v^2}}$, or (dividing top and bottom by c), $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$.
- Notice that this number is bigger than 1 (unless $v = 0$). Think of it as the time between ticks on the moving clock, measured in ticks on the stationary clock. This is a prediction called **time dilation.**

4 Thinking about more light clocks

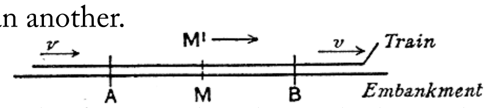
- Now let's add a second light clock, perpendicular to the first (i.e. along the floor of the rocket). Let's make it d units long, too, so the folk in the rocket see it ticking at the same rate as the first clock.
- We folk on the ground must see the two clocks ticking at the same rate, too, even though we've just inferred that the first one runs slow. In other words, the second one must run slow, by exactly the same amount. (Why? Why couldn't it just be a 'relative' matter whether the two clocks on the rocket are ticking at the same rate? *Hint:* Use a bomb.)
- Let's think about the rate of ticking of a horizontal clock of some length f. Suppose it takes T_1 seconds for light to travel along this clock, left to right. The total distance covered by the light ray is $f + vT_1$. (Why? Because vT_1 is the extra distance the right hand end of the clock moves, in T_1 seconds.) But the light ray covers a distance of cT_1 in T_1 seconds, so we know that $cT_1 = f + vT_1$. In other words, $T_1 = f/c - v$.
- We can use the same kind of reasoning for the return journey, to figure out that the light takes $T_2 = f/c+v$ to get back.
- So one complete tick of the horizontal clock takes $T_1 + T_2 = \frac{f}{c-v} + \frac{f}{c+v} = \frac{2fc}{c^2-v^2} = \frac{2f}{c}\frac{1}{1-v^2/c^2}$.
- If the horizontal clock ticks at the same rate as the vertical clock, this means that $\frac{2f}{c}\frac{1}{1-v^2/c^2} = \frac{2d}{c}\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$. So $f = d\sqrt{1-\frac{v^2}{c^2}}$. In other words, the horizontal clock looks **shorter** than the vertical clock, to the observers on the ground.

- So the horizontal clock looks **shorter** than the vertical clock, to the observers on the ground. This is **length contraction** – a second prediction of Einstein's assumption, that again falls straight out of thinking carefully about light, length and time, in the light of the assumption that Galileo has to be reconciled with Maxwell.
- Note that both **time dilation** and **length contraction** are "relative" effects. The folk on the rocket see our clocks run slow, and our lengths contract in the direction of motion, by exactly the same factors.

5 The relativity of simultaneity

- Suppose we turn our light clocks into alarm clocks, and set them to go off after one hour. Which alarm goes off first, the one on the ground or the one in the rocket?
- We say that our alarm goes off first, because their clock is running slow – but they same the same about us! So we disagree about whether one event occurs earlier or later than another.

6 Einstein on the relativity of simultaneity

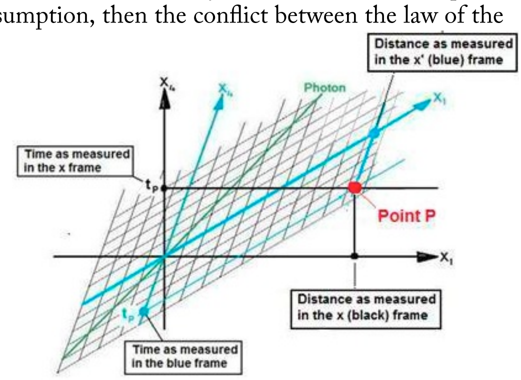


"Are two events (e.g. the two strokes of lightning A and B) which are simultaneous with reference to the railway embankment also simultaneous relatively to the train? … When we say that the lightning strokes A and B are simultaneous with respect to the embankment, we mean: the rays of light emitted at the places A and B, where the lightning occurs, meet each other at the mid-point M of the length A –> B of the embankment. But the events A and B also correspond to positions A and B on the train. Let M' be the mid-point of the distance A –> B on the travelling train. Just when the flashes of lightning occur, this point M' naturally coincides with the point M, but it moves towards the right in the diagram with the velocity v of the train. If an observer sitting in the position M' in the train did not possess this velocity, then he would remain permanently at M, and the light rays emitted by the flashes of lightning A and B would reach him simultaneously, i.e. they would meet just where he is situated. Now in reality (considered with reference to the railway embankment) he is hastening towards the beam of light coming from B, whilst he is riding on ahead of the beam of light coming from A. Hence the observer will see the beam of light emitted from B earlier than he will see that emitted from A. Observers who take the railway train as their reference-body must therefore come to the conclusion that the lightning flash B took place earlier than the lightning flash A.

We thus arrive at the important result: Events which are simultaneous with reference to the embankment are not simultaneous with respect to the train, and vice versa (relativity of simultaneity). Every reference-body (co-ordinate system) has its own particular time; unless we are told the reference-body to which the statement of time refers, there is no meaning in a statement of the time of an event.

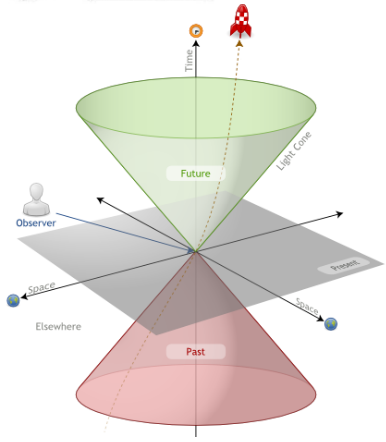
Now before the advent of the theory of relativity it had always tacitly been assumed in physics that the statement of time had an absolute significance, i.e. that it is independent of the state of motion of the body of reference. But we have just seen that this assumption is incompatible with the most natural definition of simultaneity; if we discard this assumption, then the conflict between the law of the propagation of light *in vacuo* and the principle of relativity … disappears."

- Einstein saw that just as absolute rest is an "idle cog" after Galileo, so too simultaneity is an idle cog – it has no observational significance, so can be safely left out of our picture of the world.



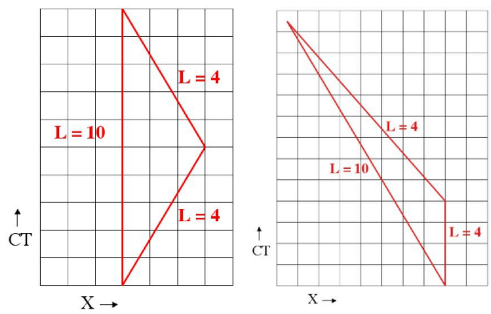
7 Not everything is relative

- '**Minkowski space**' is a useful way of representing what space and time look like in special relativity – both its 'relative' or 'frame-dependent' aspects, and the 'absolute' structure, that all (inertial) observers agree about.
- Representing time dilation and length contraction. (Top right)
- The light-cone structure – the 'absolute past', 'absolute future', and 'absolute elsewhere'. '**Worldlines**'.
- The **space-time interval**: $\Delta s^2 = c^2\Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2$. All observers agree on the total, even though they disagree about the time and space components. Added up along a worldline of a clock, it measures the elapsed time ('proper time') shown on that clock.



8 The twin paradox

- The space-time interval is **longest** along straight worldlines – which explains why a space-travelling twin will return **younger** than her Earth-bound sibling. This is the so-called 'twin paradox', though it isn't really a paradox.
- It is mistakenly thought to be a paradox because people reason that the situation is symmetric – from the perspective of the twin in the rocket it is the Earth-bound twin who 'travels'. But one twin has a straight worldline, the other doesn't, so it isn't symmetric. (The picture lower right shows the calculation of spacetime interval by the two twins – note that the answers agree.)
- If the twins travel symmetrically, in opposite directions, then there's no difference in age when they meet.



The same situation from two perspectives, but since L is invarient you get the same values. Alice experiences 10 units of time while Bob experiences 8 units (since 5^2-3^2=16=4^2).