DE BROGLIE-BOHM THEORY AND QUANTUM FIELD THEORY

WARD STRUYVE
(PERIMETER INSTITUTE)

DE BROGLIE-BOHM MODELS FOR QUANTUM FIELD THEORY:

- "DIRAC SEA MODEL"
  (JOINT WORK WITH S. COLIN)
- "MINIMALIST MODEL"
  (JOINT WORK WITH H. WESTMAN)
- "MINIMALIST MODEL + EXTRAS"
  (JOINT WORK WITH H. WESTMAN)

DE BROGLIE - BOHM APPROACH

(A.K.A. PILOT-WAVE THEORY, BOHMIAN MECHANICS, CAUSAL INTERPRETATION, ...)

- SYSTEMS ARE DESCRIBED BY
  1. WAVEFUNCTION ψ, WHICH SATISFIES SCHRODINGER EQUATION : $i\partial_t \psi = \hat{H}\psi$.
  2. EXTRA VARIABLES, CALLED BEABLES

     E.G. • PARTICLE POSITIONS : $\vec{x}_1, \dots, \vec{x}_N$
     • FIELD CONFIGURATION : $\phi(\vec{x}, t)$
     • ...

- EXAMPLE: NON-RELATIVISTIC DE BROGLIE-BOHM THEORY

  - DE BROGLIE (1926), BOHM (1952)

  - $$\begin{cases} i\partial_t \psi = \left(-\sum_{a=1}^{N} \frac{\hat{p}_a^2}{2m_a} + V \right)\psi \\ \frac{d\vec{x}_a}{dt} = \vec{v}_a = \frac{\vec{\nabla}_a S}{m_a} \end{cases} \qquad (\psi = |\psi| e^{iS})$$

- IMAGE OF THE WORLD IS IN THE BEABLES, NOT THE WAVEFUNCTION

- REPRODUCES THE STANDARD QUANTUM PREDICTIONS, PROVIDED THAT THE DISTRIBUTION ρ OF THE BEABLES EQUALS $|\psi|^2$ (= EQUILIBRIUM DISTRIBUTION)

  $\Rightarrow$ QUANTUM THEORY IS AN EFFECTIVE THEORY, DESCRIBING THE PHYSICS OF EQUILIBRIUM STATES

FOR NON-EQUILIBRIUM DISTRIBUTIONS ($\rho \neq |\psi|^2$), THE THEORY PREDICTS DEVIATIONS FROM QUANTUM THEORY

(IS BEING EXPLORED BY VALENTINI)

HOW CAN THE DE BROGLIE-BOHM VELOCITY LAW BE OBTAINED

- CONSIDER CONTINUITY EQUATION

$$\partial_t |\psi|^2 + \sum_{k=1}^{n} \vec{\nabla}_k \cdot (\vec{J}_k |\psi|^2) = 0$$

- IN STANDARD QUANTUM THEORY:

$|\psi(x,t)|^2$ = PROBABILITY DENSITY TO FIND THE PARTICLES AT x, AT TIME t.

- IN DE BROGLIE-BOHM THEORY:

$|\psi(x,t)|^2$ = PROBABILITY DENSITY FOR PARTICLES TO BE AT x, AT TIME t.

- $v = (\vec{v}_1, \ldots, \vec{v}_n)$ IS INTERPRETED AS THE VELOCITY FIELD OF THE PARTICLES

($\rightsquigarrow$ IN ANALOGY WITH FLUID DYNAMICS)

QUANTUM EQUILIBRIUM

- $\rho(x, t_0) = |\psi(x,t_0)|^2 \Rightarrow \rho(x,t) = |\psi(x,t)|^2 \ \underline{\forall t}$

  I.E. ρ PRESERVES ITS FORM AS A FUNCTIONAL OF ψ UNDER THE DBB EVOLUTION

  $\leadsto$ IS CALLED <u>EQUIVARIANCE</u>

- TECHNICAL DEFINITION:

$$\boxed{\rho^{\psi} \text{ IS EQUIVARIANT IF } \rho^{\psi}_t = \rho^{\psi_t}}$$

ρ^{ψ}_t: OBTAINED FROM ρ^{ψ} BY DBB EVOLUTION

ρ^{ψ_t}: OBTAINED FROM ρ^{ψ} BY SCHRODINGER EVOLUTION OF ψ.

- UNIQUENESS OF QUANTUM EQUILIBRIUM:

  IF $\begin{cases} \rho^{\psi} \text{ IS EQUIVARIANT} \\ \rho^{\psi}(x) = N^{\psi} \rho\,(x, \psi(x), \dots, \partial_1^{m_1} \cdots \partial_N^{m_N} \psi(x), \dots) \end{cases}$

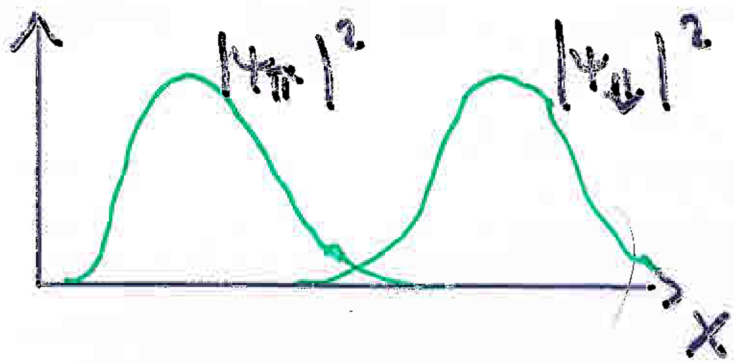
  THEN $\boxed{\rho^{\psi} = |\psi|^2}$

(GOLDSTEIN & STRUYVE 2007)

DE BROGLIE-BOHM THEORY REPRODUCES THE PREDICTIONS OF QUANTUM THEORY

- EQUILIBRIUM DISTRIBUTION $|\psi|^2$
- WAVEFUNCTIONS REPRESENTING MACROSCOPICALLY DISTINCT SYSTEMS ARE APPROXIMATELY DISJOINT, e.g.

$\psi_{\uparrow\uparrow}$, $\psi_{\downarrow\downarrow}$



$\Rightarrow$ GUARANTEES THAT MEASUREMENT OUTCOMES ARE DISPLAYED AND RECORDED IN THE BEABLE CONFIGURATIONS

AND, MORE GENERALLY, THAT THE BEABLES CONTAIN, ON THE MACROSCOPIC LEVEL, AN IMAGE OF THE EVERYDAY CLASSICAL WORLD.

• EMERGENCE OF STANDARD COLLAPSE RULE

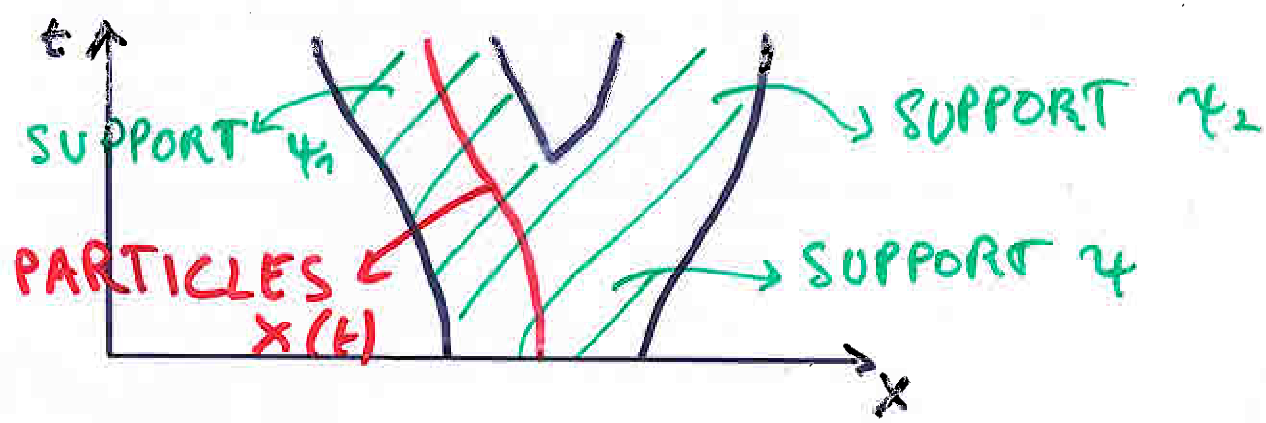
IN DEBROGLIE-BOHM THEORY WE HAVE AN "EFFECTIVE COLLAPSE":

- SCHRODINGER EVOLUTION

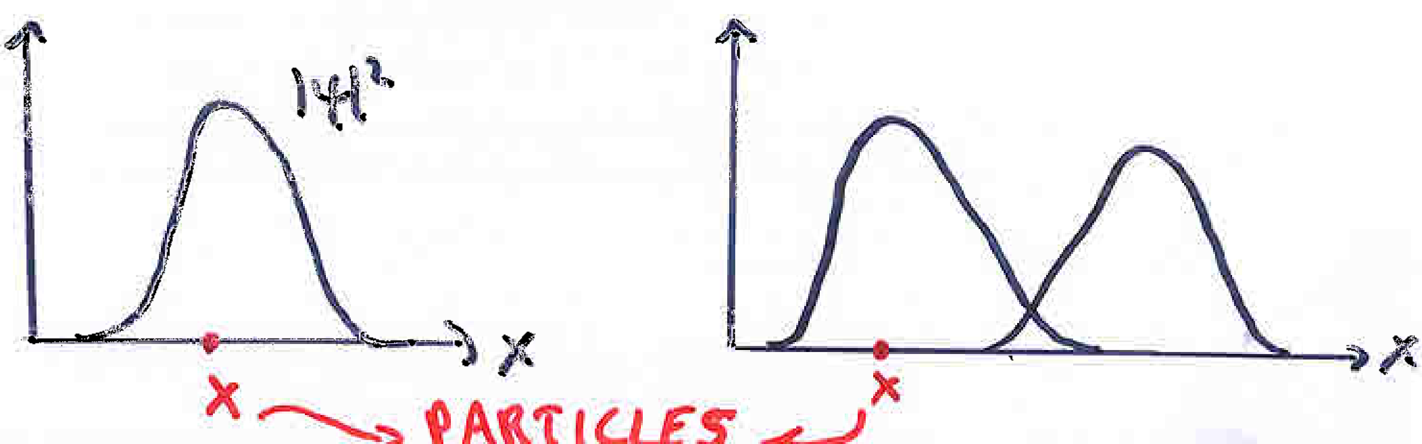
$$\psi(x) \longrightarrow \underbrace{\psi_1(x) + \psi_2(x)}_{\text{NO COMMON SUPPORT}}$$

$$(\psi_1(x)\,\psi_2(x) = 0)$$

SUPPORT OF ψ



DISTRIBUTION $|\psi|^2$



- EFFECTIVE COLLAPSE:

ψ_2 DOES NOT INFLUENCE THE MOTION OF THE CONFIGURATION $x(t)$

$$\Rightarrow \boxed{\psi_1 + \psi_2 \longrightarrow \psi_1}$$

WE HAVE AN EFFECTIVE COLLAPSE IN MEASUREMENT SITUATIONS:

$$\psi_{outcome\,1}(\vec{x})\,\psi_{\Uparrow}(y) + \psi_{outcome\,2}(\vec{x})\,\psi_{\Downarrow}(y)$$

SYSTEM UNDER OBSERVATION

APPARATUS NEEDLE

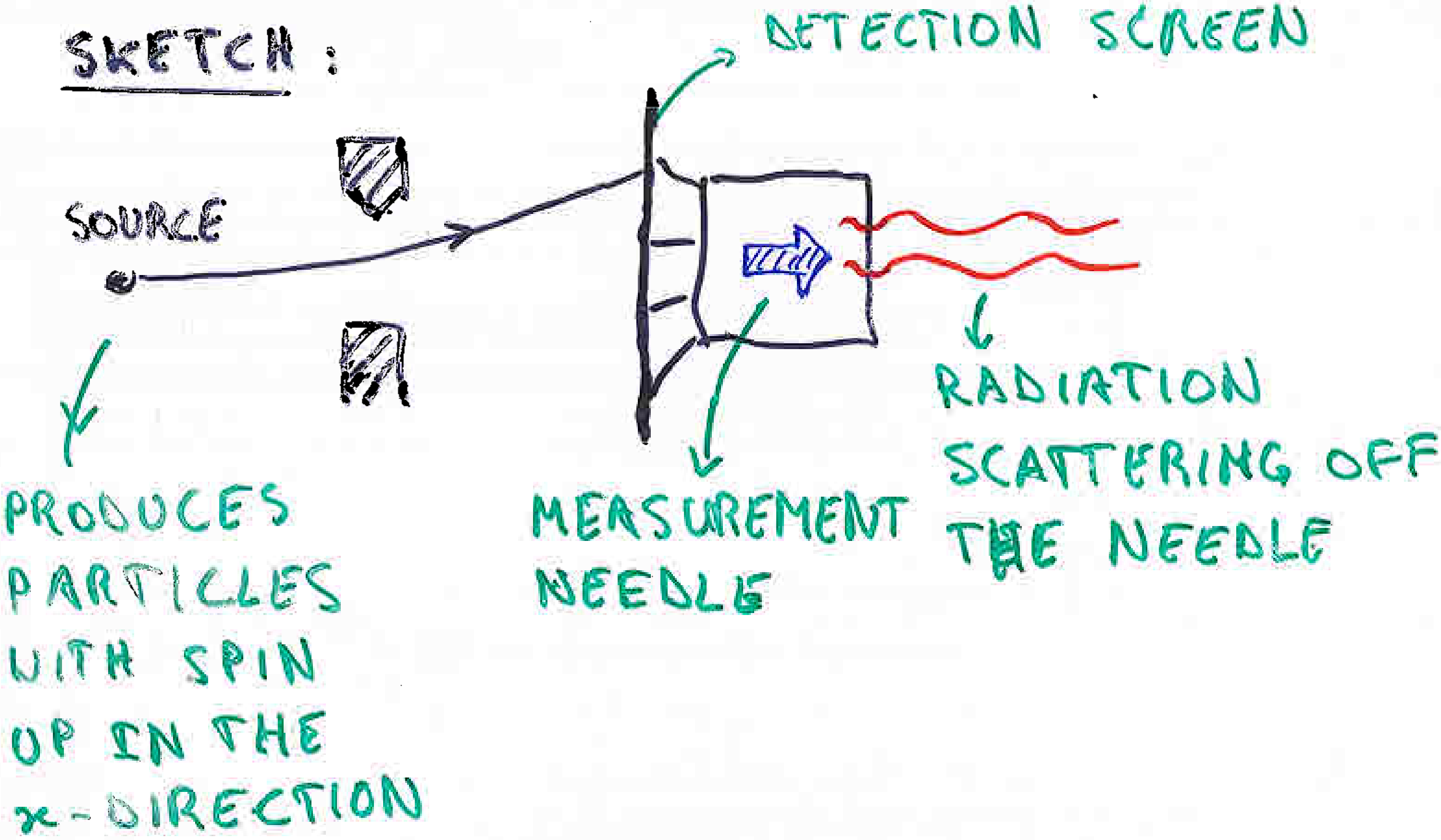
$\rightsquigarrow$ WILL BE NON-OVERLAPPING

$\Downarrow$

EFFECTIVE COLLAPSE

- STERN-GERLACH EXPERIMENT

SKETCH:



QUANTUM MECHANICAL DESCRIPTION

$$\overbrace{|+\rangle_x}^{=\frac{1}{\sqrt{2}}\left(|+\rangle_z + |-\rangle_z\right)} |\Rightarrow\rangle |\approx\rangle$$

(SCHRODINGER EVOLUTION)

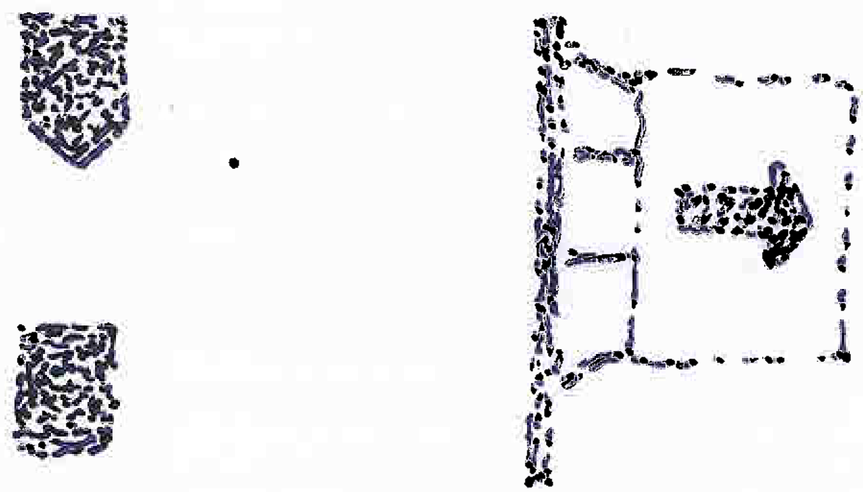
$$\longrightarrow \frac{1}{\sqrt{2}} |+\rangle_z |\Uparrow\rangle |\approx\rangle + \frac{1}{\sqrt{2}} |-\rangle_z |\Downarrow\rangle |\approx\rangle$$

(COLLAPSE)

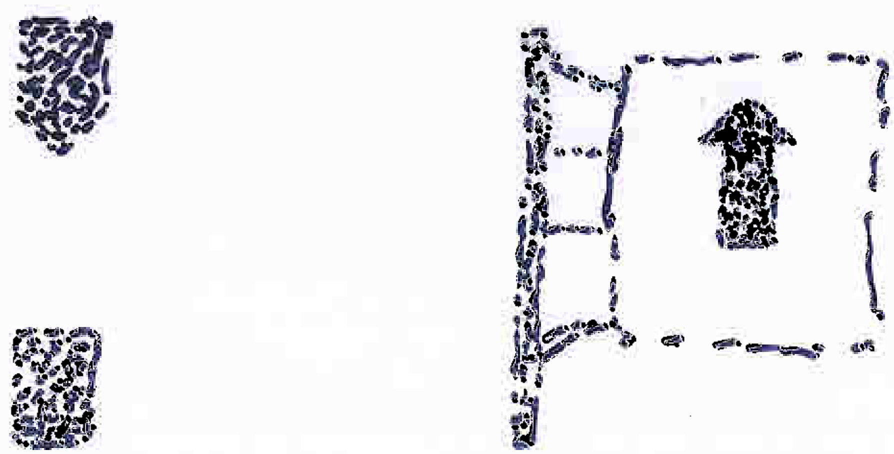
$$\longrightarrow |+\rangle_z |\Uparrow\rangle |\approx\rangle$$

BOHMIAN DESCRIPTION

AT TIME t_1



AT TIME $t_2 > t_1$



QUANTUM FIELD THEORY

- WHICH ONTOLOGY? PARTICLES? FIELDS?

- STANDARD QUANTUM FIELD THEORY IS ABOUT THE S-MATRIX, WHICH GIVES SCATTERING AMPLITUDES BETWEEN ASYMPTOTIC FREE INCOMING ($t \to -\infty$) AND OUTGOING STATES ($t \to +\infty$), NOT ABOUT THE SCHRODINGER EVOLUTION OF SOME STATE VECTOR

$\Rightarrow$ HOW TO DEFINE THE DYNAMICS OF THE BEABLES?

WE ASSUME ULTRAVIOLET MOMENTUM CUT-OFF AND BOUNDED SPACE. THIS MAKES THE TOTAL NUMBER OF DEGREES OF FREEDOM FINITE, SO THAT WE HAVE A WELL-DEFINED SCHRODINGER EVOLUTION

PARTICLE ONTOLOGY

→ FERMIONIC FIELD ON THE LATTICE

- BELL (1984)
- BEABLES : FERMION NUMBERS AT THE LATTICE POINTS
- STOCHASTIC
- CONTINUUM GENERALIZATIONS

1) DURR, GOLDSTEIN, TUMULKA, ZANGHI (2002-2005)

→ STOCHASTIC

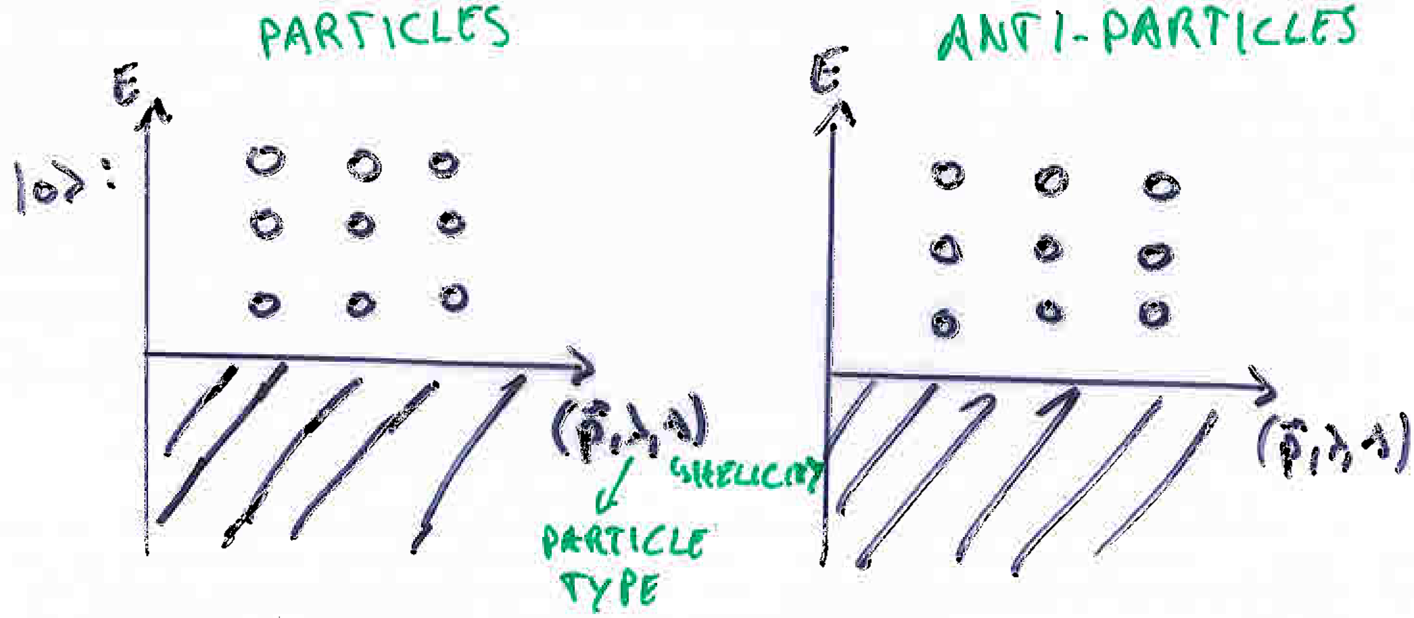
2) COLIN (2003, 2004), COLIN & STRUYVE (2007)

→ DETERMINISTIC

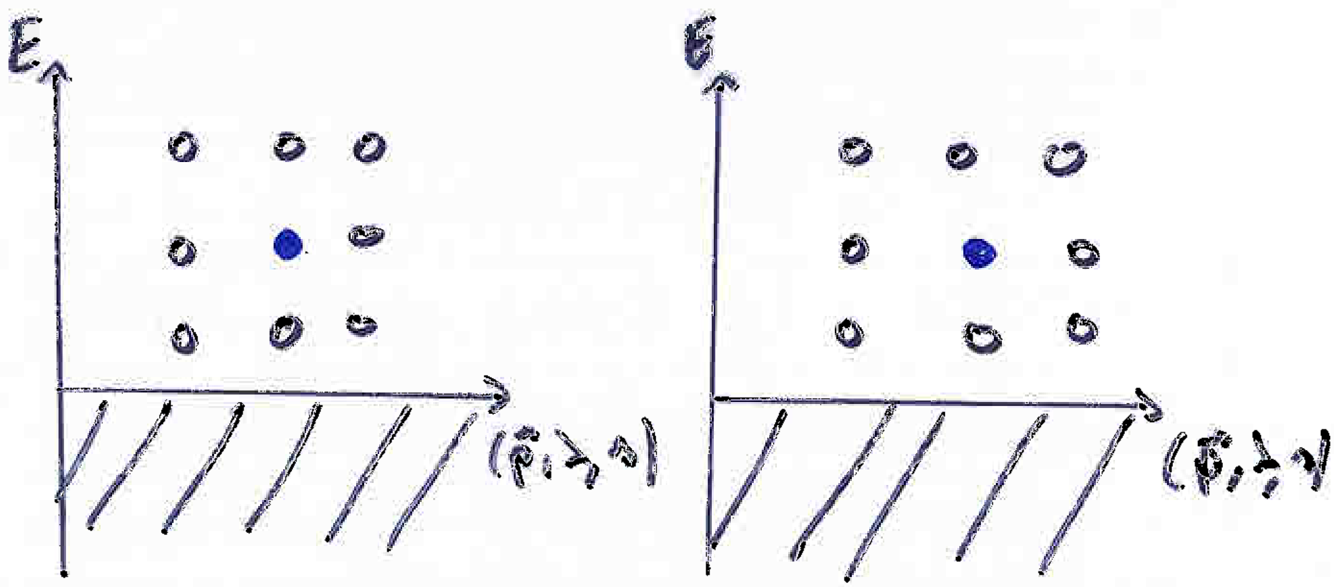
→ DIRAC SEA MODEL

PAIR CREATION

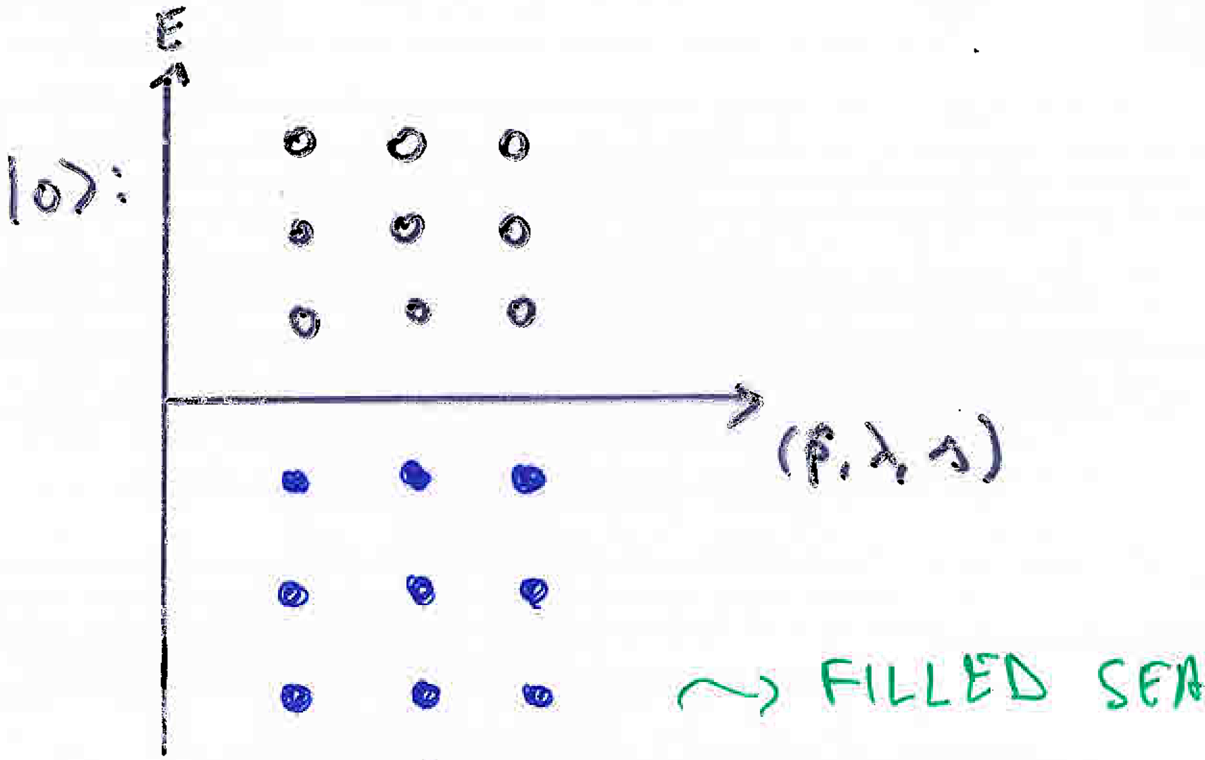
→ STANDARD REPRESENTATION



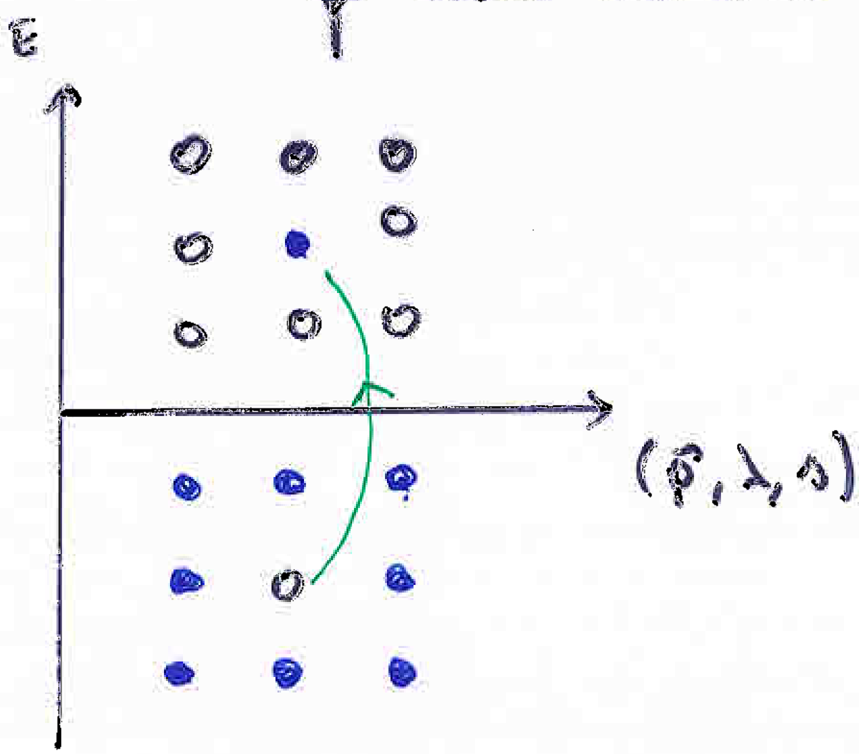
↓ PAIR CREATION



→ DIRAC SEA REPRESENTATION



↓ PAIR CREATION



→ DIRAC SEA MODEL

- WAVEFUNCTION $|\psi\rangle$ IN THE DIRAC SEA REPRESENTATION:

$$\psi_{\lambda_1 a_1 \dots \lambda_n a_n}(\vec{x}_1, \dots, \vec{x}_n) = \langle \lambda_1, a_1, \vec{x}_1; \dots, \lambda_n, a_n, \vec{x}_n | \psi \rangle$$

a → SPINOR INDEX

λ → PARTICLE TYPE

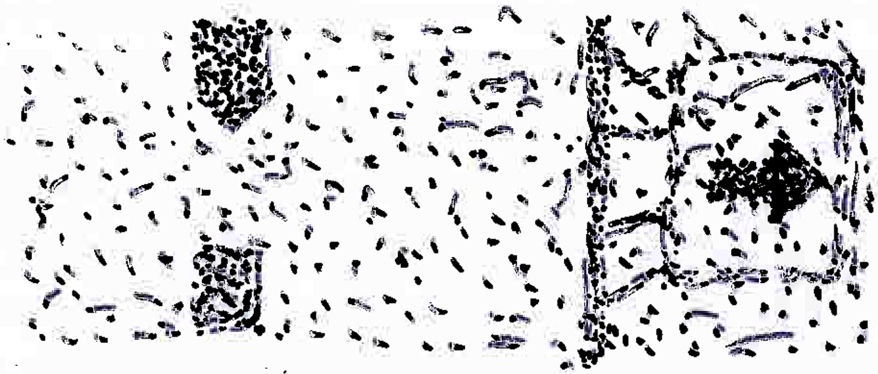
- BEABLES: PARTICLE POSITIONS $\vec{x}_1, \dots, \vec{x}_n$
- EQUILIBRIUM DISTRIBUTION:

$$\rho^{\psi}(\vec{x}_1, \dots, \vec{x}_n) = \sum_{\substack{\lambda_1 \dots \lambda_n \\ a_1 \dots a_n}} |\psi_{\lambda_1 a_1 \dots \lambda_n a_n}(\vec{x}_1, \dots, \vec{x}_n)|^2$$

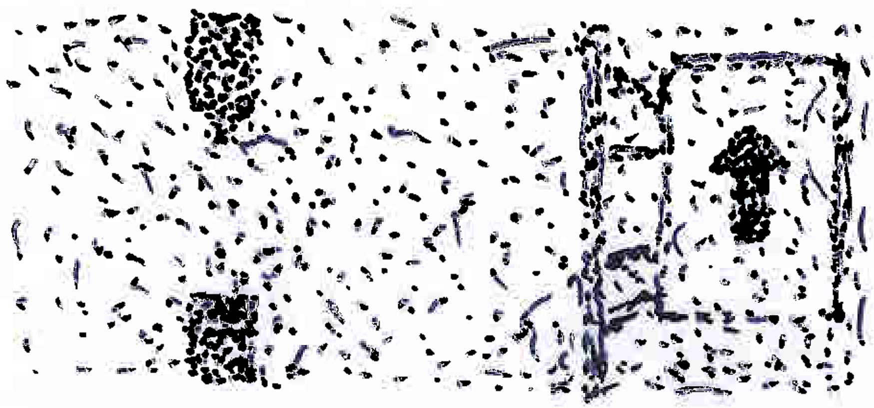
- DETERMINISTIC

STERN-GERLACH EXPERIMENT

TIME t_1



TIME $t_2 > t_1$



FIELD ONTOLOGY

→ FREE ELECTROMAGNETIC FIELD

- BOHM (1952)
- WAVEFUNCTION $|\Psi\rangle$ IN THE FIELD REPRESENTATION : $\Psi(\vec{B}) = \langle \vec{B} | \Psi \rangle$

  ↓ MAGNETIC FIELD

- BEABLE : $\vec{B}(\vec{x})$
- EQUILIBRIUM DISTRIBUTION : $|\Psi(B)|^2$
- SIMILARLY FOR OTHER BOSONIC FIELDS (E.G. SCALAR FIELD $(\Psi(\phi), \phi(\vec{x}))$)

→ FERMIONIC FIELDS

1) - HOLLAND (1988)

- WAVEFUNCTION : $\underbrace{\Psi(\alpha, \beta, \gamma)}_{\parallel}$

$(\alpha(\vec{k}), \beta(\vec{k}), \gamma(\vec{k}))$

ANGLE-VALUED FIELDS ON MOMENTUM SPACE

- BEABLE : $\psi(\vec{x})$

DEFINED IN TERMS OF

- BEABLES $(\alpha(\vec{k}), \beta(\vec{k}), \gamma(\vec{k}))$
- $\Psi(\alpha, \beta, \gamma)$

- REPRODUCES QUANTUM PREDICTIONS ????

2) - VALENTINI (1992)

- WAVEFUNCTION : $\Psi(\eta)$

ANTI-COMMUTING FIELDS

- BEABLE : $\eta(\vec{x})$

- PROBLEMATIC ! (STRUYVE 2004)

→ MINIMALIST MODEL (FOR QED)

- STRUYVE & WESTMAN (2006, 2007)

- WAVEFUNCTION

$$\Psi_f(\vec{B}) = \langle f, \vec{B} \mid \Psi \rangle$$

↳ MAGNETIC FIELD

↳ LABELS FERMIONIC DEGREES OF FREEDOM

- BEABLE : $\vec{B}(\vec{x})$

- EQUILIBRIUM DISTRIBUTION :

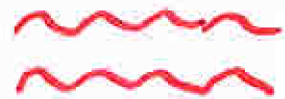
$$\rho^{\Psi}(\vec{B}) = \sum_f |\Psi_f(B)|^2$$

- NO BEABLES FOR FERMIONIC DEGREES OF FREEDOM !

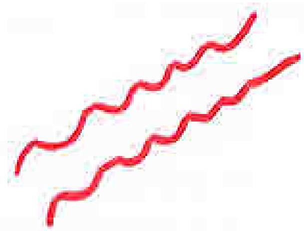
- DETERMINISTIC

STERN - GERLACH EXPERIMENT

TIME t_1



TIME t_2



→ MINIMALIST MODEL + EXTRAS

- STRUYVE & WESTMAN (2007)

- WAVE FUNCTION:

$$\Psi_f(\vec{B})$$

- BEABLES:

  - "MAGNETIC FIELD" $\vec{B}(\vec{x})$

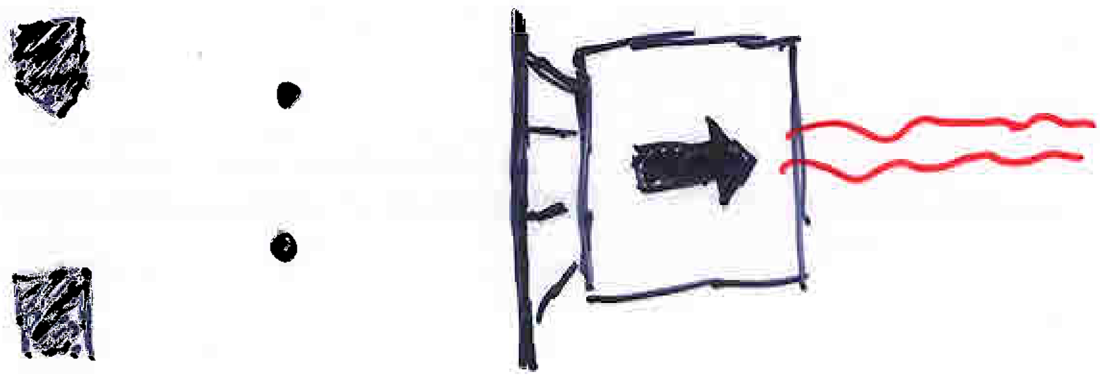
  - "CHARGE DISTRIBUTION" $\rho(\vec{x})$, WITH

$$\rho(\vec{x},t) = \left.\frac{\sum_{f,f'} \Psi_f^*(\vec{B},t)\,\hat{\rho}_{ff'}(\vec{x})\,\Psi_{f'}(\vec{B},t)}{\sum_f |\Psi_f(\vec{B},t)|^2}\right|_{\vec{B}=\underbrace{\vec{B}(t)}_{\text{FIELD BEABLE}}}$$

$$\hat{\rho}_{ff'}(\vec{x}) = \langle f|\underbrace{\hat{\rho}(\vec{x})}_{=\text{ CHARGE DENSITY OPERATOR}}|f'\rangle$$

STERN-GERLACH EXPERIMENT

TIME t_1



TIME $t_2 > t_1$

CONCLUSION + OUTLOOK

- SEVERAL DE BROGLIE-BOHM MODELS FOR QUANTUM FIELD THEORY
  - PARTICLE ONTOLOGY, FIELD ONTOLOGY
  - DETERMINISTIC, STOCHASTIC

- CAN REGULATORS BE REMOVED WHILE PRESERVING A WELL-DEFINED DYNAMICS FOR THE BEABLES ?

- STUDY OF NON-EQUILIBRIUM
  → MIGHT BE FOUND UNDER "EXTREME CIRCUMSTANCES" (VALENTINI)
  E.G. STRONG GRAVITATIONAL FIELDS, INFLATION, ...
  ⇝ REQUIRES FIELD THEORETICAL DESCRIPTION