Should we think of quantum probabilities as Bayesian probabilities?

Carlton M. Caves

C. M. Caves, C. A. Fuchs, R. Schack, "Subjective probability and quantum certainty,"
Studies in History and Philosophy of Modern Physics **38**, 255--274 (2007)..

Department of Physics and Astronomy
University of New Mexico
and
Department of Physics
University of Queensland

caves@info.phys.unm.edu
http://info.phys.unm.edu/~caves

Perimeter Institute-Australia Foundations Workshop
Sydney, 2008 February 3

Yes, because facts never determine probabilities or quantum states.

Subjective Bayesian probabilities

Category distinction

Facts

Outcomes of *events*
Truth values of *propositions*

Objective

Probabilities

Agent's *degree of belief*
in outcome of an event or
truth of a proposition

Subjective

Facts never imply probabilities.

Two agents in possession of the same facts can assign different probabilities.

Subjective Bayesian probabilities

Probabilities

Agent's *degree of belief* in outcome of an event or truth of a proposition.

Consequence of ignorance

Agent's *betting odds*

Subjective

Rules for manipulating probabilities are *objective* consequences of consistent betting behavior (Dutch book).

Subjective Bayesian probabilities

Facts in the form of observed data *d* are used to update probabilities via Bayes's rule:

conditional (model, likelihood)

$$\underbrace{p(h|d)}_{\text{posterior}} = \frac{\overbrace{p(d|h)}^{\text{conditional (model, likelihood)}}\ \overbrace{p(h)}^{\text{prior}}}{p(d)}$$

prior

posterior

The posterior always depends on the prior, *except* when *d* logically implies h_0:

$$\mathrm{Pr}(d|h) = 0 \text{ for } h \neq h_0 \quad \Longrightarrow \quad \mathrm{Pr}(h_0|d) = 1\ .$$

Objective probabilities

- Logical probabilities (objective Bayesian): symmetry implies probability
  - Symmetries are applied to judgments, not to facts.

- Probabilities as frequencies: probability as verifiable fact
  - Bigger sample space; exchangeability.
  - Frequencies are facts, not probabilities.

~~QM: Derivation of quantum probability rule from infinite frequencies?~~

C. M. Caves, R. Schack, ``Properties of the frequency operator do not imply the quantum probability postulate,'' Annals of Physics **315**, 123-146 (2005) [Corrigendum: **321**, 504--505 (2006)].

- Objective chance (propensity): probability as specified fact
  - Some probabilities are ignorance probabilities, but others are specified by the facts of a "chance situation."
  - Specification of "chance situation": same, but different.

objective *chance*

QM: Probabilities from physical law. Salvation of objective chance?

	Classical (realistic, deterministic) world		Quantum world	
State space	Simplex of probabilities for microstates		Convex set of density operators	
State	Extreme point Microstate	Ensemble	Extreme point Pure state State vector	Ensemble Mixed state Density operator
	x_j	$p(x_j)$	$\lvert\psi\rangle$	$\rho = \sum_j p_j \lvert\psi_j\rangle\langle\psi_j\rvert$

Scorecard:

1. Predictions for fine-grained measurements
2. Verification (state determination)
3. State change on measurement
4. Uniqueness of ensembles
5. Nonlocal state change (steering)
6. Specification (state preparation)

Objective	**Subjective**	**Objective**	**Subjective**

	Classical (realistic, deterministic) world		Quantum world	
State space	Simplex of probabilities for microstates		Convex set of density operators	
State	Extreme point Microstate	Ensemble	Extreme point Pure state State vector	Ensemble Mixed state Density operator
Fine-grained measurement	Certainty	Probabilities	Certainty or Probabilities	Probabilities
	x_j	$p(x_j)$	$\|\psi\rangle$	$\rho = \sum_j p_j \|\psi_j\rangle\langle\psi_j\|$

Certainty:

Orthonormal measurement basis that contains $|\psi\rangle$.

Objective	**Subjective**	Objective	**Subjective**

	Classical (realistic, deterministic) world		Quantum world	
State space	Simplex of probabilities for microstates		Convex set of density operators	
State	Extreme point Microstate	Ensemble	Extreme point Pure state State vector	Ensemble Mixed state Density operator
Verification: state determination	Yes	No	No	No

Whom do you ask for the system state? The system or an agent?

Objective	**Subjective**	Ubjective	**Subjective**

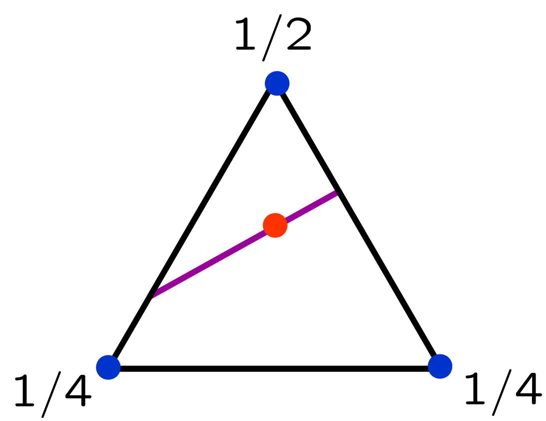
	Classical (realistic, deterministic) world		Quantum world	
State space	Simplex of probabilities for microstates		Convex set of density operators	
State	Extreme point Microstate	Ensemble	Extreme point Pure state State vector	Ensemble Mixed state Density operator
State change on measurement	No	Yes	Yes	Yes

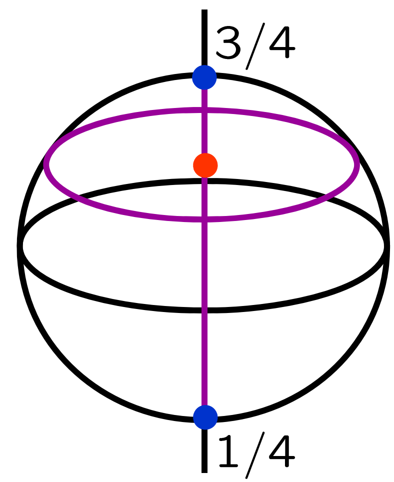
State-vector reduction or wave-function collapse

Real physical disturbance?

Objective	**Subjective**	Ubjective	**Subjective**

	Classical (realistic, deterministic) world		Quantum world	
State space	Simplex of probabilities for microstates		Convex set of density operators	
State	Extreme point Microstate	Ensemble	Extreme point Pure state State vector	Ensemble Mixed state Density operator
Uniqueness of ensembles	Yes	No	No	No





Objective	**Subjective**	Ubjective	**Subjective**

	Classical (realistic, deterministic) world		Quantum world	
State space	**Simplex of probabilities for microstates**		**Convex set of density operators**	
State	Extreme point Microstate	Ensemble	Extreme point Pure state State vector	Ensemble Mixed state Density operator
Nonlocal state change (steering)	No	Yes	Yes	Yes

$$p_0 = 1/2 \qquad p_1 = 1/2$$

$$p_{0|0} = 3/4 \qquad p_{0|1} = 1/4$$
$$p_{1|0} = 1/4 \qquad p_{1|1} = 3/4$$

$$\begin{aligned} |\Psi\rangle &= \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \\ &= \frac{1}{\sqrt{2}}(|\mathbf{n}, -\mathbf{n}\rangle - |-\mathbf{n}, \mathbf{n}\rangle \end{aligned}$$

Real nonlocal physical disturbance?

	Objective	**Subjective**	Subjective	**Subjective**

	Classical (realistic, deterministic) world		Quantum world	
State space	Simplex of probabilities for microstates		Convex set of density operators	
State	Extreme point Microstate	Ensemble	Extreme point Pure state State vector	Ensemble Mixed state Density operator
Specification: state preparation	Yes	No	Copenhagen: Yes	Copenhagen: Yes

Copenhagen interpretation: Classical facts specifying the properties of the preparation device determine a pure state.

Copenhagen (objective preparations view) becomes the home of objective chance, with nonlocal physical disturbances

Objective	**Subjective**	**Objective**	**Objective**

Copenhagen	Classical (realistic, deterministic) world		Quantum world	
State space	Simplex of probabilities for microstates		Convex set of density operators	
State	Extreme point Microstate	Ensemble	Extreme point Pure state State vector	Ensemble Mixed state Density operator
Fine-grained measurement	Certainty	Probabilities	Certainty or Probabilities	Probabilities
Verification: state determination	Yes	No	No	No
State change on measurement	No	Yes	Yes	Yes
Uniqueness of ensembles	Yes	No	No	No
Nonlocal state change (steering)	No	Yes	Yes	Yes
Specification: state preparation	Yes	No	Yes	Yes
	Objective	**Subjective**	**Objective**	**Objective**

Classical and quantum updating

Facts in the form of observed data *d* are used to update probabilities via Bayes's rule:

$$\underbrace{p(h|d)}_{\text{posterior}} = \frac{\overbrace{p(d|h)}^{\text{conditional (model, likelihood)}}\;\overbrace{p(h)}^{\text{prior}}}{p(d)}$$

The posterior always depends on the prior, *except* when *d* logically implies h_0:

$$\Pr(d|h) = 0 \text{ for } h \neq h_0$$
$$\Longrightarrow \quad \Pr(h_0|d) = 1\,.$$

Facts in the form of observed data *d* are used to update quantum states:

$$\underbrace{\rho_d}_{\text{posterior}} = \frac{\overbrace{\mathcal{A}_d}^{\text{quantum operation (model)}}(\overbrace{\rho}^{\text{prior}})}{p(d)}$$

Quantum state preparation:

ρ_d does not depend on ρ.

The posterior state *always* depends on prior beliefs, *even* for quantum state preparation, because there is a judgment involved in choosing the quantum operation.

Facts never determine probabilities or quantum states.

Where does Copenhagen go wrong?

The Copenhagen interpretation forgets that the preparation device is quantum mechanical. A detailed description of the operation of a preparation device (provably) involves prior judgments in the form of quantum state assignments.

Subjective Bayesian	Classical (realistic, deterministic) world		Quantum world	
State space	Simplex of probabilities for microstates		Convex set of density operators	
State	Extreme point Microstate	Ensemble	Extreme point Pure state State vector	Ensemble Mixed state Density operator
Fine-grained measurement	Certainty	Probabilities	Certainty or Probabilities	Probabilities
Verification: state determination	Yes	No	No	No
State change on measurement	No	Yes	Yes	Yes
Uniqueness of ensembles	Yes	No	No	No
Nonlocal state change (steering)	No	Yes	Yes	Yes
Specification: state preparation	Yes	No	No	No
	Objective	**Subjective**	**Subjective**	**Subjective**

Is a quantum coin toss more random than a classical one? Why trust a quantum random generator over a classical one?

$$|\psi\rangle = |\uparrow\rangle = (|\rightarrow\rangle + |\leftarrow\rangle)/\sqrt{2}$$

Measure spin along *z* axis: $p_\uparrow = 1$ $p_\downarrow = 0$

Measure spin along *x* axis: $p_\rightarrow = 1/2$ $p_\leftarrow = 1/2$

C. M. Caves, R. Schack, "Quantum randomness," in preparation.

quantum coin toss

	Classical (realistic, deterministic) world		Quantum world	
State space	**Simplex of probabilities for microstates**		**Convex set of density operators**	
State	Extreme point Microstate	Ensemble	Extreme point Pure state State vector	Ensemble Mixed state Density operator
Fine-grained measurement	Certainty	Probabilities	**Certainty or Probabilities**	Probabilities

Is a quantum coin toss more random than a classical one? Why trust a quantum random generator over a classical one?

$$|\psi\rangle = |{\uparrow}\rangle = (|{\rightarrow}\rangle + |{\rightarrow}\rangle)/\sqrt{2}$$

Measure spin along *z* axis: $p_{\uparrow} = 1$ $p_{\downarrow} = 0$

Measure spin along *x* axis: $p_{\rightarrow} = 1/2$ $p_{\leftarrow} = 1/2$

quantum coin toss

Standard answer: The quantum coin toss is objective, with probabilities guaranteed by physical law.

Subjective Bayesian answer? No inside information.

Pure states and inside information

Party *B* has *inside information* about event *E,* relative to party *A*, if *A* is willing to agree to a bet on *E* that *B* believes to be a sure win. *B* has *one-way inside information* if *B* has inside information relative to *A*, but *A* does not have any inside information relative to *A*.

The unique situation in which *no other party can have one-way inside information* relative to a party *Z* is when *Z* assigns a pure state. *Z* is said to have a *maximal belief structure.*

Subjective Bayesian answer

We trust quantum over classical coin tossing because an insider attack on classical coin tossing can never be ruled out, whereas the beliefs that lead to a pure-state assignment are inconsistent with any other party's being able to launch an insider attack.

Taking a stab at ontology

CMC only

Quantum systems are defined by *attributes*, such as position, momentum, angular momentum, and energy or Hamiltonian. These attributes—and thus the numerical particulars of their eigenvalues and eigenfunctions and their inner products—are *objective* properties of the system.

The *value* assumed by an attribute is not an objective property, and the *quantum state* that we use to describe the system is purely subjective.

Taking a stab at ontology

1. The attributes orient and give structure to a system's Hilbert space. Without them we are clueless as to how to manipulate and interact with a system.
2. The attributes are unchanging properties of a system, which can be determined from facts. The attributes determine the structure of the world.
3. The Hamiltonian orients a system's Hilbert space now with the same space later.
4. Convex combinations of Hamiltonian evolutions are essentially unique (up to degeneracies).

Why should you care?
If you do care, how can this be made convincing?
Status of quantum operations?
Effective attributes and effective Hamiltonians? "Effective reality"?