Logical Constants, Sequent Structures and Speech Acts

the case of modal operators

Greg Restall

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logical constants

defining rules

making it explicit

easy cases

$\wedge \quad \vee \quad \sim \quad \rightarrow \quad \forall \quad \exists$

simple modality

2D modality

the upshot

logical constants

what is a logical constant?

it’s **logical**

it’s **constant**

it’s **subject matter independent**

it's **definable**

it plays a **structural role** in deduction logical consequence is **necessary**

there are **paradigm cases**,

like $\wedge$, $\vee$, $\rightarrow$, $\sim$, $\forall$ and $\exists$

what about modality?

boundary drawing

valid

invalid

models

proofs

proofs

models

Tarski's models,

Kripke structures

Montague semantics

etc.

Logical constants are **structure invariant**

models

proofs

Natural deduction,
sequent calculus

logical constants have **good proof-theoretical rules**

proofs

soundness no overlap

models

proofs

completeness no gap

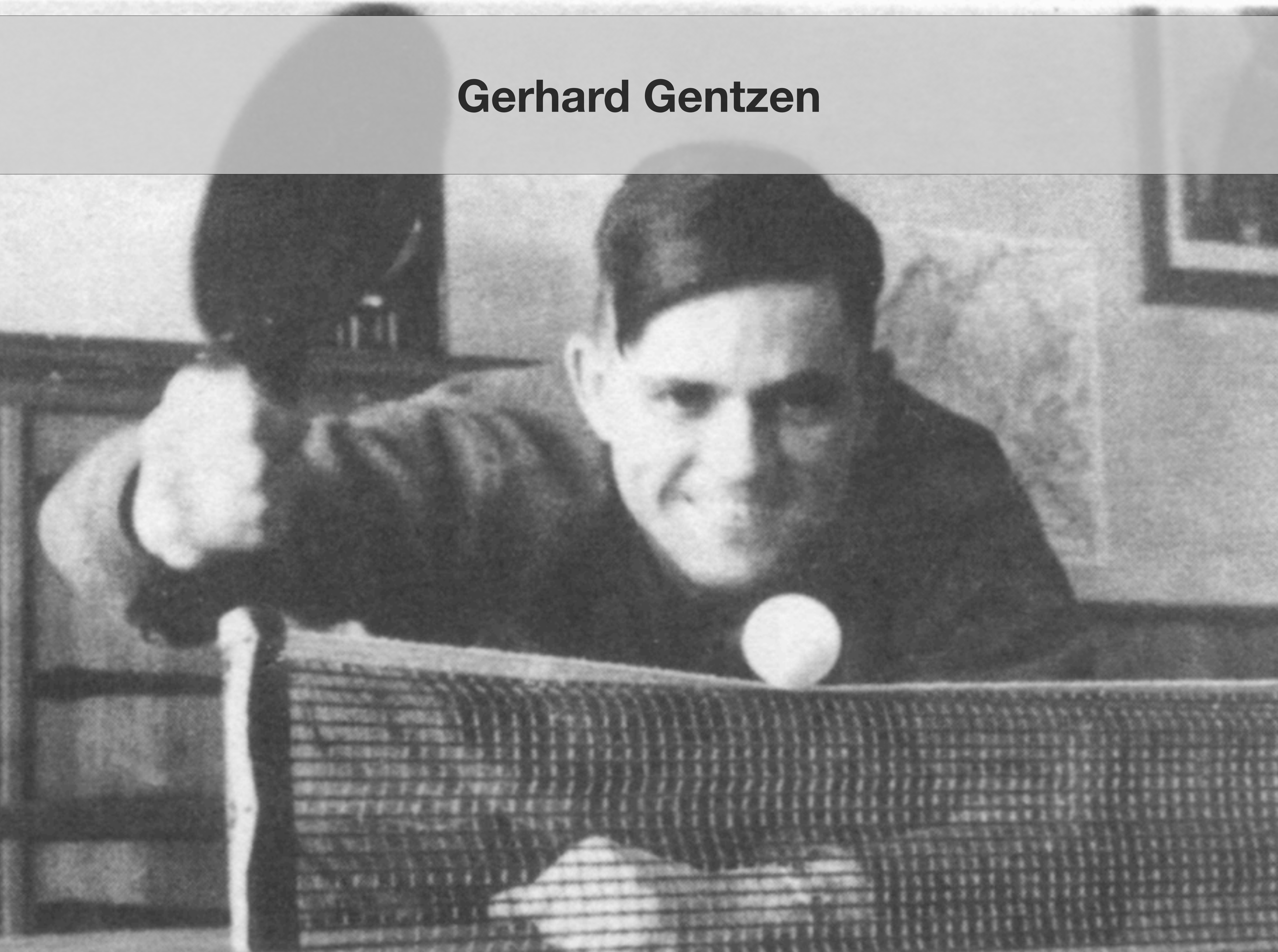
models

proofs and **models**

proofs and **models**
can both play a role
in **semantics**

proofs are good for accounts connected with use…

Gerhard Gentzen



Die Schlußfiguren-Schemata.

1.21. Schemata für Struktur-Schlußfiguren:

Verdünnung:

im Antezedens: $\dfrac{\Gamma \to \Theta}{\mathfrak{D}, \Gamma \to \Theta}$, im Sukzedens: $\dfrac{\Gamma \to \Theta}{\Gamma \to \Theta, \mathfrak{D}}$;

Zusammenziehung:

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1.22. Schemata für Logische-Zeichen-Schlußfiguren:

UES: $\dfrac{\Gamma \to \Theta, \mathfrak{A} \quad \Gamma \to \Theta, \mathfrak{B}}{\Gamma \to \Theta, \mathfrak{A} \& \mathfrak{B}}$

UEA: $\dfrac{\mathfrak{A}, \Gamma \to \Theta}{\mathfrak{A} \& \mathfrak{B}, \Gamma \to \Theta} \quad \dfrac{\mathfrak{B}, \Gamma \to \Theta}{\mathfrak{A} \& \mathfrak{B}, \Gamma \to \Theta}$

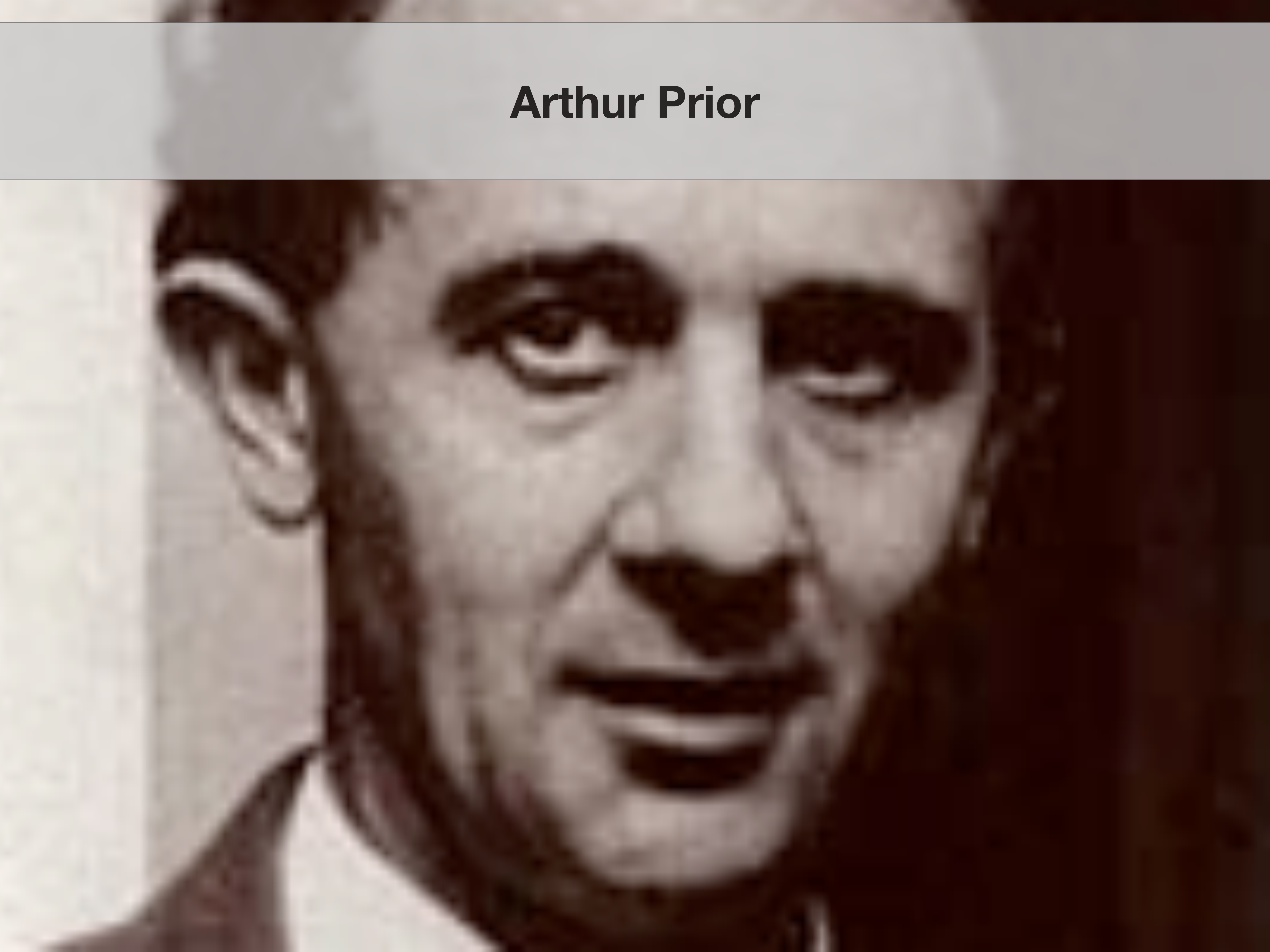
AES: $\dfrac{\Gamma \to \Theta, \mathfrak{F}\mathfrak{a}}{\Gamma \to \Theta, \forall \mathfrak{x} \mathfrak{F}\mathfrak{x}}$

OEA: $\dfrac{\mathfrak{A}, \Gamma \to \Theta \quad \mathfrak{B}, \Gamma \to \Theta}{\mathfrak{A} \vee \mathfrak{B}, \Gamma \to \Theta}$

OES: $\dfrac{\Gamma \to \Theta, \mathfrak{A}}{\Gamma \to \Theta, \mathfrak{A} \vee \mathfrak{B}} \quad \dfrac{\Gamma \to \Theta, \mathfrak{B}}{\Gamma \to \Theta, \mathfrak{A} \vee \mathfrak{B}}$

EEA: $\dfrac{\mathfrak{F}\mathfrak{a}, \Gamma \to \Theta}{\exists \mathfrak{x} \mathfrak{F}\mathfrak{x}, \Gamma \to \Theta}$

defining rules


Arthur Prior

THE RUNABOUT INFERENCE-TICKET

By A. N. Prior

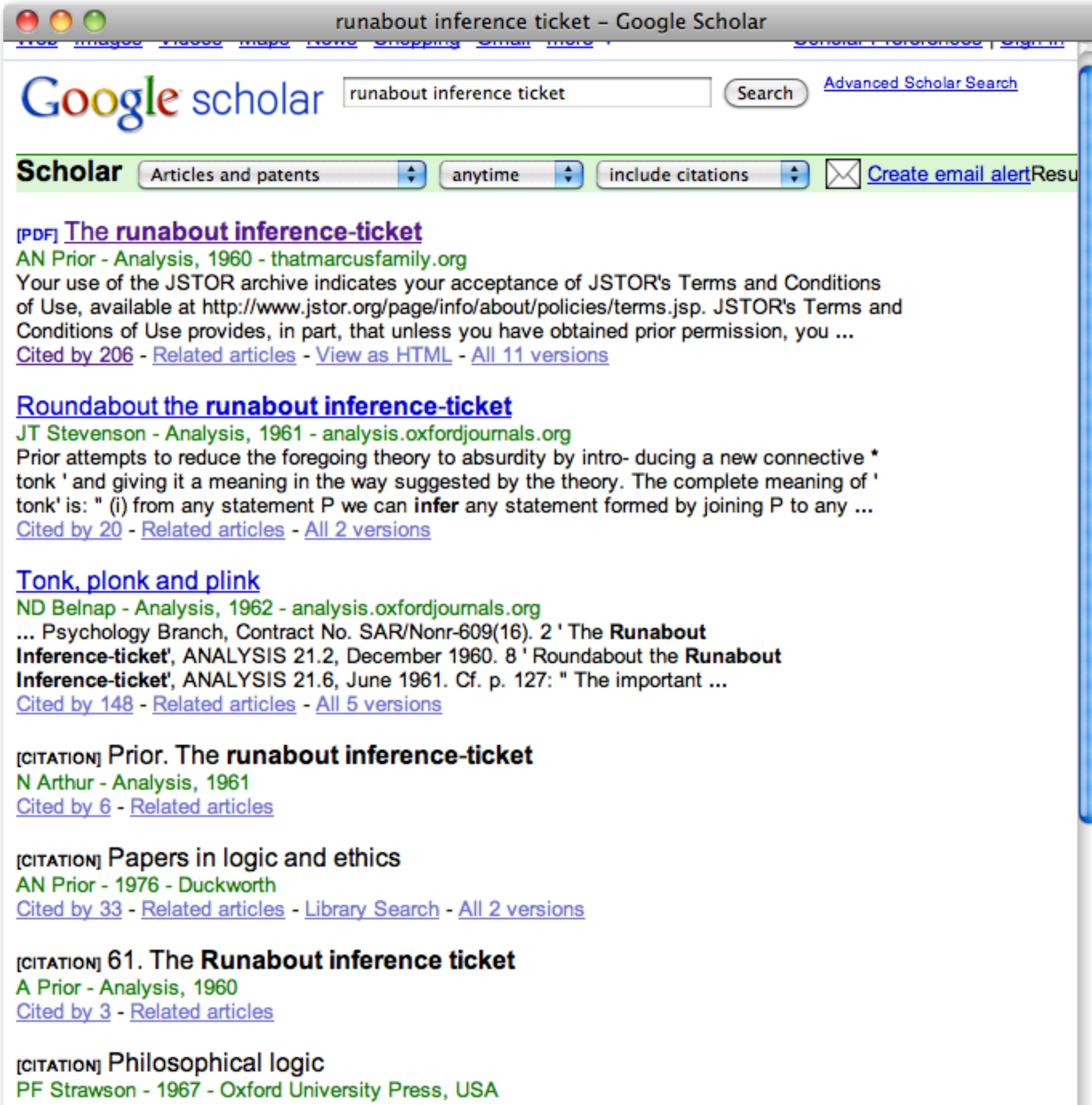
IT is sometimes alleged that there are inferences whose validity arises solely from the meanings of certain expressions occurring in them. The precise technicalities employed are not important, but let us say that such inferences, if any such there be, are analytically valid.

One sort of inference which is sometimes said to be in this sense analytically valid is the passage from a conjunction to either of its conjuncts, e.g., the inference ' Grass is green and the sky is blue, therefore grass is green '. The validity of this inference is said to arise solely from the meaning of the word ' and '. For if we are asked what is the meaning of the word ' and ', at least in the purely conjunctive sense (as opposed to, e.g., its colloquial use to mean ' and then '), the answer is said to be *completely* given by saying that (i) from any pair of statements P and Q we can infer the statement formed by joining P to Q by ' and ' (which statement we hereafter describe as ' the statement P-and-Q '), that (ii) from any conjunctive statement P-and-Q we can infer P, and (iii) from P-and-Q we can always infer Q. Anyone who has learnt to perform these inferences knows the meaning of ' and ', for there is simply nothing more *to* knowing the meaning of ' and ' than being able to perform these inferences.

A doubt might be raised as to whether it is really the case that, for any pair of statements P and Q, there is always a statement R such that

$$\frac{A \qquad B}{A \wedge B}\,[\wedge I] \qquad\qquad \frac{A \wedge B}{A}\,[\wedge E_1] \quad \frac{A \wedge B}{B}\,[\wedge E_2]$$

$$\frac{A}{A \text{ tonk } B}\,[\mathrm{tonk}I] \qquad \frac{A \text{ tonk } B}{B}\,[\mathrm{tonk}E]$$



runabout inference ticket – Google Scholar

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[PDF] The **runabout inference-ticket**
AN Prior - Analysis, 1960 - thatmarcusfamily.org
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Cited by 206 - Related articles - View as HTML - All 11 versions

Roundabout the **runabout inference-ticket**
JT Stevenson - Analysis, 1961 - analysis.oxfordjournals.org
Prior attempts to reduce the foregoing theory to absurdity by intro- ducing a new connective ' tonk ' and giving it a meaning in the way suggested by the theory. The complete meaning of ' tonk' is: " (i) from any statement P we can **infer** any statement formed by joining P to any ...
Cited by 20 - Related articles - All 2 versions

Tonk, plonk and plink
ND Belnap - Analysis, 1962 - analysis.oxfordjournals.org
... Psychology Branch, Contract No. SAR/Nonr-609(16). 2 ' The **Runabout Inference-ticket**', ANALYSIS 21.2, December 1960. 8 ' Roundabout the **Runabout Inference-ticket**', ANALYSIS 21.6, June 1961. Cf. p. 127: " The important ...
Cited by 148 - Related articles - All 5 versions

[CITATION] Prior. The **runabout inference-ticket**
N Arthur - Analysis, 1961
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AN Prior - 1976 - Duckworth
Cited by 33 - Related articles - Library Search - All 2 versions

[CITATION] 61. The **Runabout inference ticket**
A Prior - Analysis, 1960
Cited by 3 - Related articles

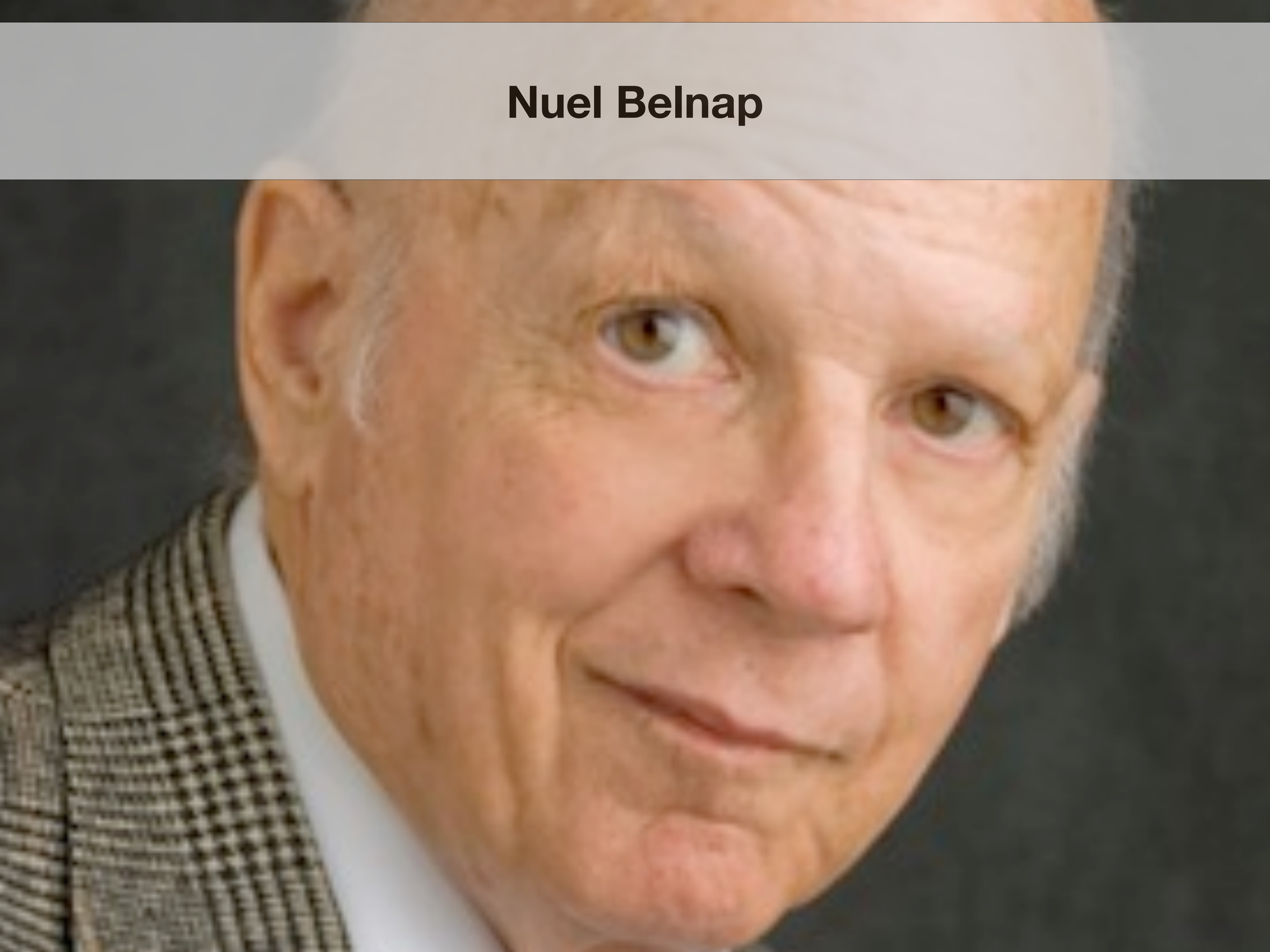
[CITATION] Philosophical logic
PF Strawson - 1967 - Oxford University Press, USA

conjunction has a **truth table**,
and tonk doesn't

— J. T. Stevenson

this uses **models**

Nuel Belnap



TONK, PLONK AND PLINK[1]

By Nuel D. Belnap

A. N. PRIOR has recently discussed[2] the connective *tonk*, where *tonk* is defined by specifying the role it plays in inference. Prior characterizes the role of *tonk* in inference by describing how it behaves as conclusion, and as premiss: (1) A $\vdash$ A-*tonk*-B, and (2) A-*tonk-B* $\vdash$ B (where we have used the sign ' $\vdash$ ' for deducibility). We are then led by the transitivity of deducibility to the validity of A $\vdash$ B, " which promises to banish *falsche Spitzfindigkeit* from Logic for ever."

A possible moral to be drawn is that connectives cannot be defined in terms of deducibility at all; that, for instance, it is illegitimate to define *and* as that connective such that (1) A-*and*-B $\vdash$ A, (2) A-*and*-B $\vdash$ B, and (3) A, B $\vdash$ A-*and*-B. We must first, so the moral goes, have a notion of what *and* means, independently of the role it plays as premiss and as conclusion. Truth-tables are one way of specifying this antecedent meaning; this seems to be the moral drawn by J. T. Stevenson.[3] There are good reasons, however, for defending the legitimacy of defining connections in terms of the roles they play in deductions.

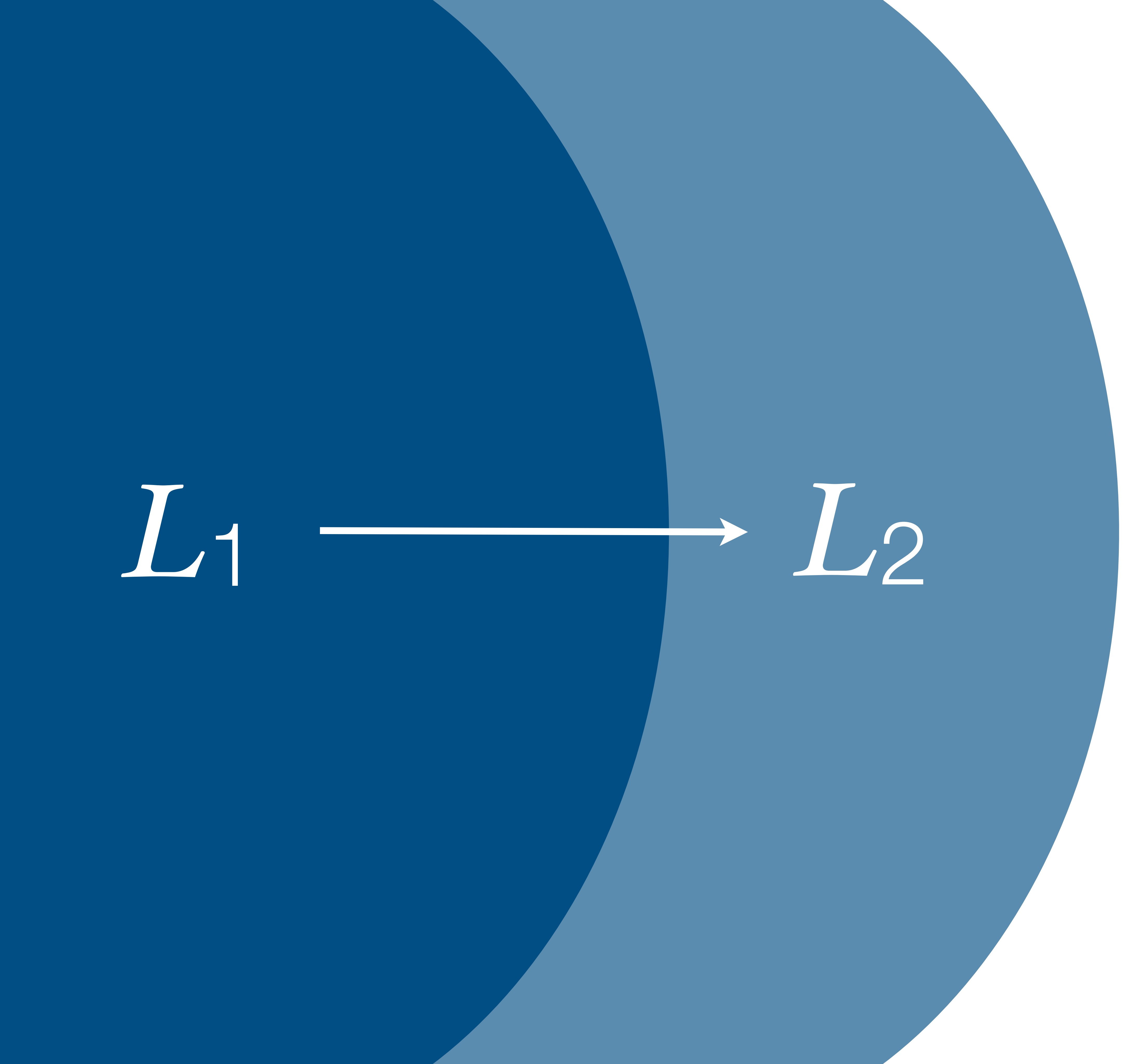
It seems plain that throughout the whole texture of philosophy one can distinguish two modes of explanation: the analytic mode, which tends to explain wholes in terms of parts, and the synthetic mode, which explains parts in terms of the wholes or contexts in which they occur.[4] In logic, the analytic mode would be represented by Aristotle, who commences with terms as the ultimate atoms, explains propositions or judgments by means of these, syllogisms by means of the propositions which go to make them up, and finally ends with the notion of a science as a tissue of syllogisms. The analytic mode is also represented by the contemporary logician who first explains the meaning of complex sentences, by means of truth-tables, as a function of their parts, and then proceeds to give an account of correct inference in terms of the sentences

conjunction is **conservative**,
and **uniquely defined**
and tonk isn't

— Nuel Belnap

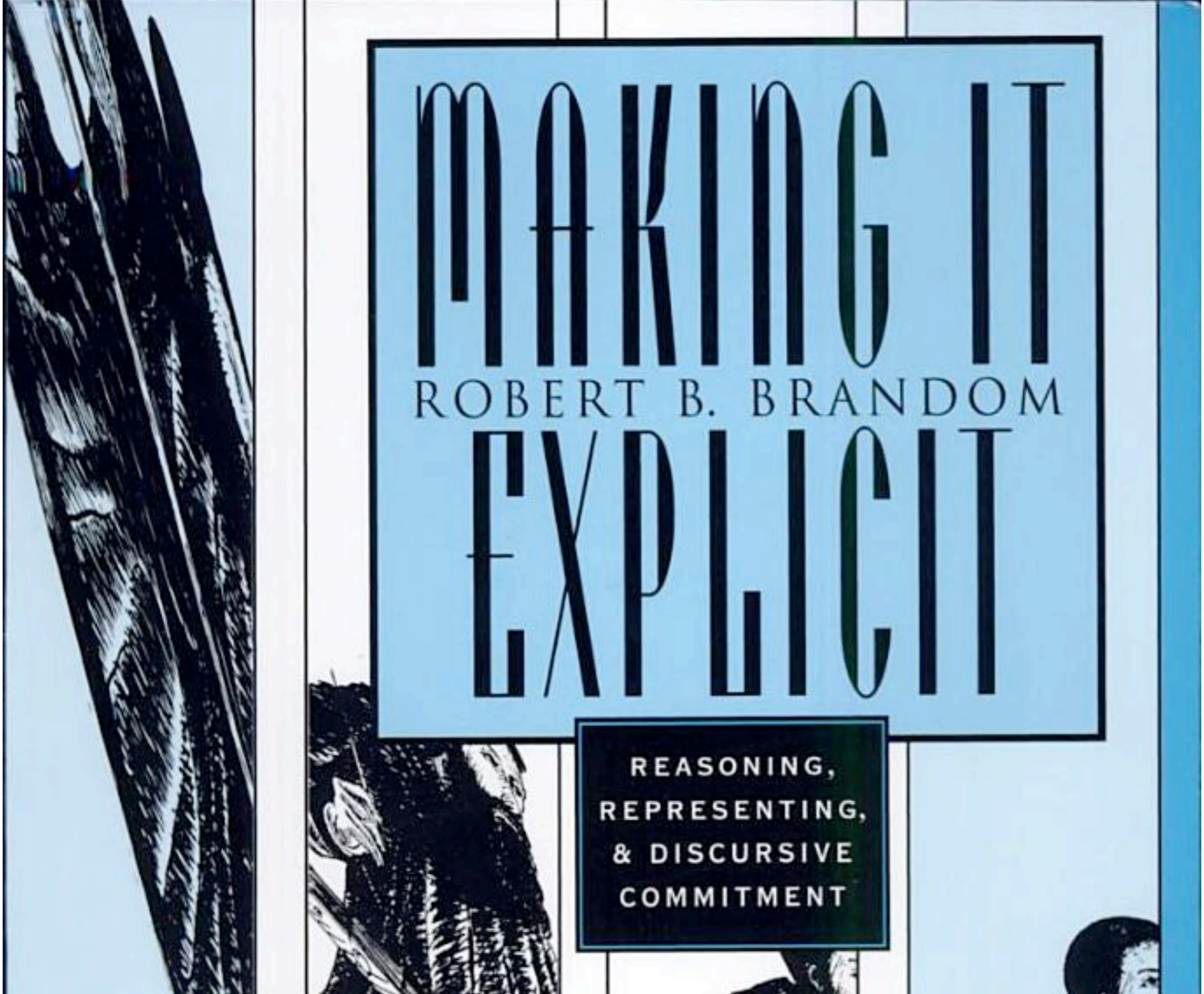
this uses **proofs**

but it’s **relative to your starting point**


L_1
L_2

Is there an **absolute notion**?

making it explicit


MAKING IT
ROBERT B. BRANDOM
EXPLICIT
REASONING,
REPRESENTING,
& DISCURSIVE
COMMITMENT

$$\frac{A \qquad B}{A \wedge B}\,[\wedge I] \qquad\qquad \frac{A \wedge B}{A}\,[\wedge E_1] \qquad \frac{A \wedge B}{B}\,[\wedge E_2]$$

$$\frac{X, A, B \vdash Y}{X, A \wedge B \vdash Y}\,[\wedge L] \qquad \frac{X \vdash A, Y \quad X' \vdash B, Y'}{X, X' \vdash A \wedge B, Y, Y'}\,[\wedge R]$$

$$\frac{X, A, B \vdash Y}{X, A \wedge B \vdash Y}\ [\wedge Df]$$

logical notions have definitions
that make explicit in language
what is implicit in discourse

easy cases

$\wedge \quad \vee \quad \sim \quad \longrightarrow \quad \forall \quad \exists$

$$\frac{X, A, B \vdash Y}{X, A \wedge B \vdash Y}\ [\wedge Df]$$

$$\frac{X \vdash A, B, Y}{X \vdash A \vee B, Y}\;[\vee Df]$$

$$\frac{X \vdash A, Y}{X, \neg A \vdash Y}\ [\neg Df]$$

$$\frac{X, A \vdash B, Y}{X \vdash A \supset B, Y}\ [\supset Df]$$

$$\frac{X \vdash A(n), Y}{X \vdash \forall x A(x), Y}\ [\forall Df]$$

(where n is free in X, Y)

$$\frac{X, A(n) \vdash Y}{X, \exists x A(x) \vdash Y} \; [\exists Df]$$

(where n is free in X, Y)

a **proof** from X to A shows us why it would be a mistake to assert each X and deny A

Multiple Conclusions

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version 1.1

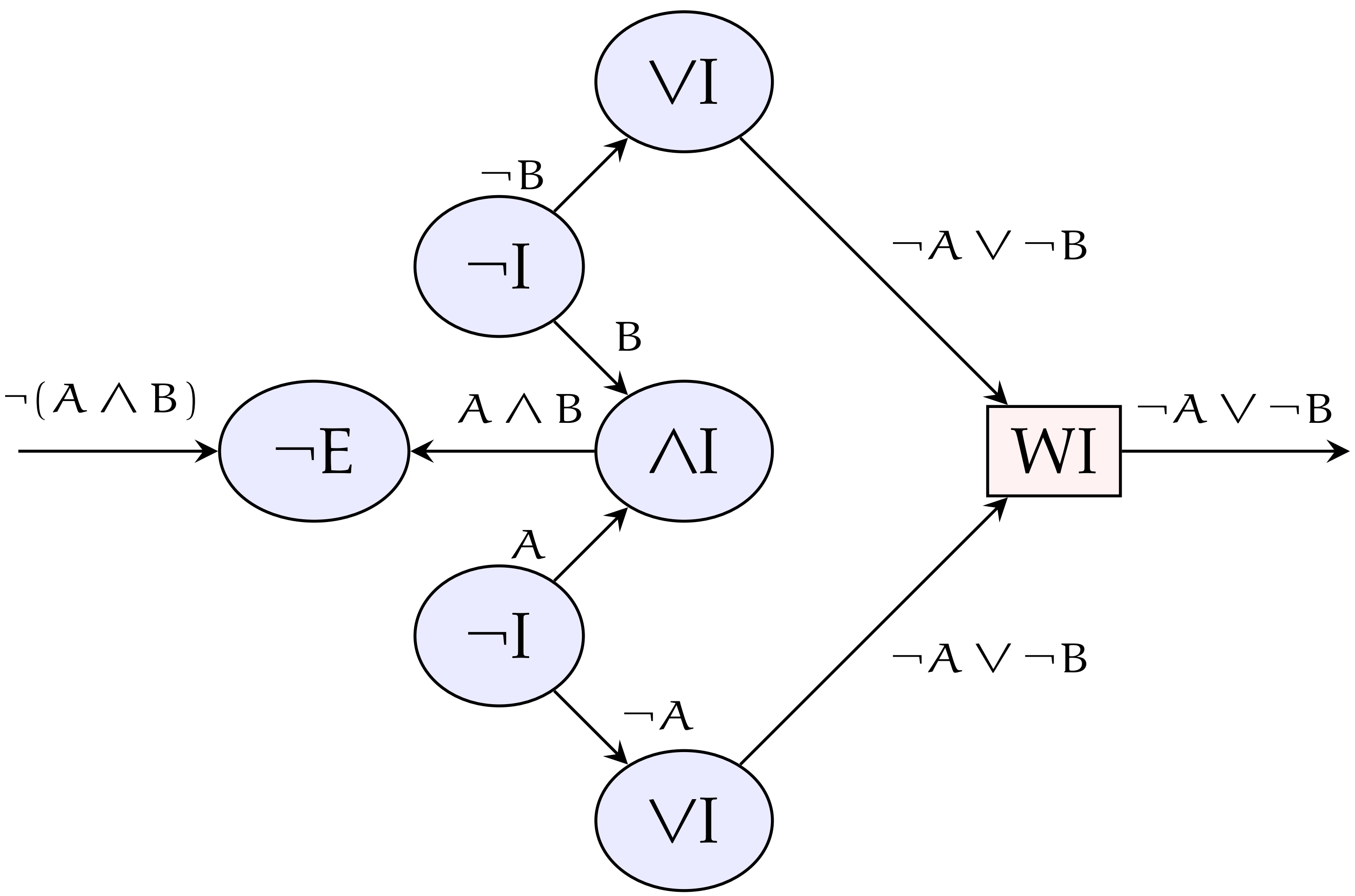
March 19, 2004

Abstract: I argue for the following four theses. (1) Denial is not to be analysed as the assertion of a negation. (2) Given the concepts of assertion and denial, we have the resources to analyse logical consequence as relating arguments with *multiple* premises and *multiple* conclusions. Gentzen's multiple conclusion calculus can be understood in a straightforward, motivated, non-question-begging way. (3) If a broadly anti-realist or inferentialist justification of a logical system works, it works just as well for *classical* logic as it does for *intuitionistic* logic. The special case for an anti-realist justification of intuitionistic logic over and above a justification of classical logic relies on an unjustified assumption about the shape of proofs. Finally, (4) this picture of logical consequence provides a relatively neutral shared vocabulary which can help us understand and adjudicate debates between proponents of classical and non-classical logics.

* * *

Our topic is the notion of logical consequence: the link between premises and conclusions, the glue that holds together deductively valid argument. How can we understand this relation between premises and conclusions? It seems that any account begs questions. Painting with very broad brushtrokes, we can sketch the landscape of disagreement like this: "Realists" prefer an analysis of logical consequence in terms of the preservation of *truth* [29]. "Anti-realists" take this to be unhelpful and offer alternative analyses. Some, like Dummett, look to preservation of *warrant to assert* [9, 36]. Others, like Brandom [5], don't define validity in terms a **proof** from X to Y shows us why it would be a mistake to assert each X and deny each Y

proofs articulate norms governing assertion and denial a discourse has a **score** keeping track of what is asserted and denied the defining rules show how we can **score** *new moves* in the assertion/denial practice, in terms of the old moves



$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{A \vdash A}{\vdash A, \neg A}}{\vdash A, \neg A \vee \neg B} \qquad \dfrac{\dfrac{A \vdash A}{\vdash B, \neg B}}{\vdash B, \neg A \vee \neg B}}{\vdash A \wedge B, \neg A \vee \neg B, \neg A \vee \neg B}}{\neg(A \wedge B) \vdash \neg A \vee \neg B, \neg A \vee \neg B}}{\neg(A \wedge B) \vdash \neg A \vee \neg B}$$

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UEA: $\mathfrak{A}, \Gamma \rightarrow \Theta$ $\mathfrak{B}, \Gamma \rightarrow \Theta$ OES: $\Gamma \rightarrow \Theta, \mathfrak{A}$ $\Gamma \rightarrow \Theta, \mathfrak{B}$

Structural rules give us constraints on scores

AES: $\dfrac{\Gamma \rightarrow \Theta, \mathfrak{F}a}{\Gamma \rightarrow \Theta, \forall \mathfrak{x} \mathfrak{F}\mathfrak{x}}$ EEA: $\dfrac{\mathfrak{F}a, \Gamma \rightarrow \Theta}{\exists \mathfrak{x} \mathfrak{F}\mathfrak{x}, \Gamma \rightarrow \Theta}$

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connective and quantifier rules make some implicit aspect of the scoresheet explicit in assertion/denial

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EEA: $\dfrac{\mathfrak{F}\mathfrak{a}, \Gamma \to \Theta}{\exists \mathfrak{x} \mathfrak{F}\mathfrak{x}, \Gamma \to \Theta}$

this account connects semantics with **normative pragmatics** the logic/non-logic boundary is determined by **two choices**

the **structural context**,
given by the space of **scores**

and the choice of
vocabulary,
given that context

simple modality

a **proof** from X to Y shows us how a position in which each X is asserted and each Y is denied is out of bounds

assertion and denial
needn’t be **flat**

I can assert or deny

under a supposition

an assertion of “**I’m in Sydney**” clashes with its denial.

an assertion of “**I’m in Sydney**” doesn’t clash with denying “**I’m in Sydney**” under the scope of “**suppose I couldn’t get here**.”

modal discourse
is filled with shifts like these

why not take this into account
in scoring discourse?

$X \vdash \Upsilon$ tells us that it'd be a mistake to assert X and deny Υ

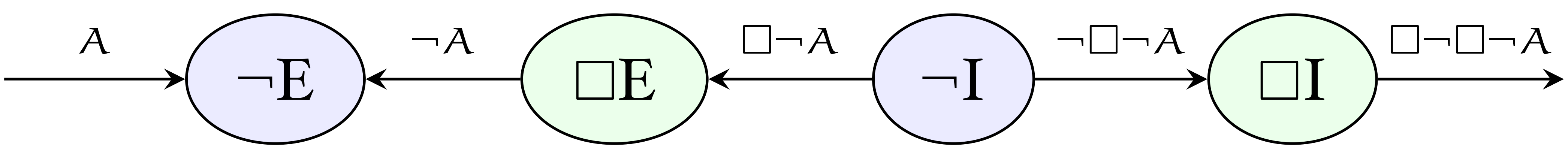
$X \vdash \Upsilon \mid U \vdash V$ tells us that it'd be a mistake to assert X and deny Υ (in one part of the discourse) and to assert U and deny V (in another).

hypersequents suit proof systems for modal logics

$$\frac{\mathcal{H}[\,\mathsf{X} \vdash \Box A, \mathsf{Y}\,]}{\mathcal{H}[\;\vdash A \;\mid\; \mathsf{X} \vdash \mathsf{Y}\,]}\;[\Box Df]$$

$$\frac{\mathcal{H}[\mathsf{X} \vdash \mathsf{Y} \mid \mathsf{X}', A \vdash \mathsf{Y}']}{\mathcal{H}[\mathsf{X}, \Box A \vdash \mathsf{Y} \mid \mathsf{X}' \vdash \mathsf{Y}']}\ [\Box L] \qquad \frac{\mathcal{H}[\ \vdash A \mid \mathsf{X} \vdash \mathsf{Y}]}{\mathcal{H}[\mathsf{X} \vdash \Box A, \mathsf{Y}]}\ [\Box R]$$

$$\dfrac{\dfrac{\dfrac{\dfrac{A \vdash A}{\neg A, A \vdash}\,[\neg\mathrm{L}]}{A \vdash \;\mid\; \Box\neg A \vdash}\,[\Box\mathrm{L}]}{A \vdash \;\mid\; \vdash \neg\Box\neg A}\,[\neg\mathrm{R}]}{A \vdash \;\mid\; \vdash \Box\neg\Box\neg A}\,[\Box\mathrm{R}]$$



$$\frac{\mathcal{H}[\, \mathsf{X} \vdash \mathsf{Y} \;\mid\; \mathsf{X}', A \vdash_{@} \mathsf{Y}' \,]}{\mathcal{H}[\, \mathsf{X}, @A \vdash \mathsf{Y} \;\mid\; \mathsf{X}' \vdash_{@} \mathsf{Y}' \,]} \; [@Df]$$

2D modality

there are two different kinds of **shift**

indicative
(suppose I’m wrong)

and **subjunctive**
(suppose things go differently)

suppose Oswald **didn’t** shoot JFK


SPECIAL COMMEMORATIVE SECTION

The New York Times.

LATE CITY EDITION

KENNEDY IS KILLED BY SNIPER AS HE RIDES IN CAR IN DALLAS; JOHNSON SWORN IN ON PLANE

TEXAN ASKS UNITY

LEFTIST ACCUSED

Gov. Connally Shot; Mrs. Kennedy Safe

Why America Weeps

PRESIDENT'S BODY WILL LIE IN STATE

PARTIES' OUTLOOK FOR '64 CONFUSED

suppose Oswald **hadn’t** shot JFK

Mark Lance

Philosophers' Imprint

volume 7, no. 4
may 2007

STEREOSCOPIC VISION:

Persons, Freedom, and Two Spaces of Material Inference

Mark Lance
Georgetown University

W. Heath White
University of North Carolina at Wilmington

Freedom, oh freedom, well that's just some people talkin'.
— The Eagles

What is a person, as opposed to a non-person? One might begin to address the question by appealing to a second distinction: between *agents*, characterized by the ability to act freely and intentionally, and mere patients, caught up in events but in no sense authors of the happenings involving them. An alternative way to address the question appeals to a third distinction: between *subjects* — bearers of rights and responsibilities, commitments and entitlements, makers of claims, thinkers of thoughts, issuers of orders, and posers of questions — and mere objects, graspable or evaluable by subjects but not themselves graspers or evaluators.

We take it as a methodological point of departure that these three distinctions are largely coextensive, indeed coextensive in conceptually central cases. Granted, these distinctions can come apart. One might think that 'person' applies to anything that is worthy of a distinctive sort of moral respect and think this applicable to some fetuses or the deeply infirm elderly. Even if the particular respect due such beings is importantly different from "what we owe each other", such respect could still be thought to be of the kind distinctively due people, and think this even while holding that such people lack agentive or subjective capacity. Similarly, one might think dogs or various severely impaired humans to be attenuated subjects but not agents.

Without taking any particular stand on such examples, our methodological hypothesis is that such cases, if they exist, are understood as persons (agents, subjects) essentially by reference to paradigm cases and, indeed, to a single paradigm within which person/non-person, subject/object, and agent/patient are conceptually connected.[1] Stated

1. For one detailed development of this sort of paradigm-riff structure, and a defense of the possibility of concepts essentially governed by such a structure, see Lance and Little (2004). Discussions with Hilda Lindeman have helped

indicative and **subjunctive** shifts are independently motivated for creatures who act on the basis of their views

this structure grounds
a system for a 2D modal logic

for **necessity** (subjunctive)

a priori knowability (indicative)

& **actuality** (interacts with both)

$$\cfrac{\cfrac{a = b \vdash \quad | \quad Fa \vdash Fb}{a = b \vdash \quad | \quad \vdash Fa \supset Fb}\,[\supset Df]}{a = b \vdash \Box(Fa \supset Fb)}\,[\Box Df]$$

good

$$\cfrac{\cfrac{a = b \vdash \quad \| \quad Fa \vdash Fb}{a = b \vdash \quad \| \quad \vdash Fa \supset Fb}\,[\supset Df]}{a = b \vdash \mathrm{APK}\,(Fa \supset Fb)}\,[\mathrm{APK}\,Df]$$

bad

$$\dfrac{\dfrac{a = b \vdash \quad | \quad Fa \vdash Fb}{a = b \vdash \quad | \quad \vdash Fa \supset Fb}\,[\supset Df]}{a = b \vdash \Box(Fa \supset Fb)}\,[\Box Df]$$

good

$$\dfrac{\dfrac{\cancel{a = b \vdash \quad \| \quad Fa \vdash Fb}}{a = b \vdash \quad \| \quad \vdash Fa \supset Fb}\,[\supset Df]}{a = b \vdash \mathrm{APK}\,(Fa \supset Fb)}\,[\mathrm{APK}\,Df]$$

bad

$$
\begin{array}{c l}
\vdash \mid \ p \vdash_{@} p & \\
\hline
\vdash \mid \ p \vdash_{@} @p & [@Df] \\
\hline
\vdash \mid \vdash_{@} p \supset @p & [\supset Df] \\
\hline
\vdash \Box(p \supset @p) & [\Box Df]
\end{array}
$$

bad

$$
\begin{array}{c l}
\vdash \parallel \ p \vdash_{@} p & \\
\hline
\vdash \parallel \ p \vdash_{@} @p & [@Df] \\
\hline
\vdash \parallel \vdash_{@} p \supset @p & [\supset Df] \\
\hline
\vdash \mathrm{APK}\,(p \supset @p) & [\mathrm{APK}\ Df]
\end{array}
$$

good

$$\dfrac{\dfrac{\dfrac{\vdash \mid\ p \vdash_{@} p}{\vdash \mid\ p \vdash_{@} @p}\,[@Df]}{\vdash \mid \vdash_{@} p \supset @p}\,[\supset Df]}{\vdash \Box(p \supset @p)}\,[\Box Df]$$

bad

$$\dfrac{\dfrac{\dfrac{\vdash \parallel\ p \vdash_{@} p}{\vdash \parallel\ p \vdash_{@} @p}\,[@Df]}{\vdash \parallel \vdash_{@} p \supset @p}\,[\supset Df]}{\vdash \mathrm{APK}\,(p \supset @p)}\,[\mathrm{APK}\,Df]$$

good

A CUT-FREE SEQUENT SYSTEM FOR TWO-DIMENSIONAL MODAL LOGIC, AND WHY IT MATTERS

Greg Restall*

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NOVEMBER 22, 2010

Version 0.99

Abstract: The two-dimensional modal logic of Davies and Humberstone [3] is an important aid to our understanding the relationship between *actuality*, *necessity* and *a priori knowability*. I show how a cut-free hypersequent calculus for 2D modal logic not only captures the logic precisely, but may be used to address issues in the epistemology and metaphysics of our modal concepts. I will explain how use of our concepts motivates the inference rules of the sequent calculus, and then show that the completeness of the calculus for Davies–Humberstone models explains why those concepts have the structure described by those models. The result is yet another application of the completeness theorem.

MOTIVATION

The 'two-dimensional modal logic' of Davies and Humberstone [3] is an important aid to our understanding the relationship between *actuality*, *necessity* and

*Thanks to audiences at the weekly Logic Seminar and the weekly Philosophy Seminar at the University of Melbourne, as well as presentations at the 2008 Australasian Association of Philosophy Annual Conference, and seminars at Carnegie Mellon, Connecticut, Dubrovnik, Leipzig, Macquarie, Pittsburgh and St Andrews—including Conrad Asmus, Jc Beall, Nuel Belnap, John Bigelow, David Chalmers, Simon D'Alfonso, Bogdan Dicher, Anil Gupta, Allen Hazen, Lloyd Humberstone, Hannes Leitgeb, Peter Menzies, Graham Priest, Laura Schroeter, Shawn Standefer and Heinrich Wansing—for helpful feedback on this material over the last

the upshot

these rules are **conservative**
and **uniquely defining**

if we agree on what indicative and subjunctive shifts occur in a discourse then we **coordinate** on these modal concepts

(we can coordinate on the meaning of $\Box$ without agreeing on whether or not a particular necessity claim is true)

(after all, we can coordinate on the meaning of $\wedge$ without agreeing on whether or not a particular conjunction is true)

these modal concepts
arise freely from the
stratified structure
of our discourse

and the rules show how
these modal concepts
are **grounded**
in our capacities

the rules tell us how to **reason** with these modal concepts

and so, can play a role
in **modal epistemology**

the general structure of completeness theorems (idealise invalid sequents) gives us something to say about **possible worlds** too

and so, this can play a role
in **modal ontology**

thank you!